Dealing with Conditional Probability Computations: a More Advanced Examples

Next week:

- I will give out prizes.
- I bring in lots of dice, and we'll do a huge dice experiment.

Now recall:

Example:

Suppose we have 2 jars, named Jar I and Jar II. Jar I contains 10 green jelly beans and 5 purple jelly beans, and Jar II contains 4 jelly beans of each color.

We draw out one jelly bean at random from Jar I, and then put it in Jar II. We then mix up Jar II and draw one jelly bean at random from Jar II.

And also recall:

Challenge problem: In last week's jelly bean example (the original version, not the one in the homework), find

P(the bean you get from II was originally in I | the bean you get from II is purple)

Hint: Say you have three numbers, a, b and c. Then if $a \times b = c$, then $b = c \div a$.

How do we do the challenge problem?

Once again, let's do an easier problem first:

> Find
>
> P(the bean you get from I was purple | the bean you get from II is purple)

In other words, suppose I didn't look at the bean I got from jar I when I put it into jar 2. Now I look at the bean I draw from jar II, and I see it is purple. What are the chances that the bean I drew from jar I was purple?

To this, recall

Handy Rule #1:

> Say A and B are two "events" involving an experiment. Then
>
> P(A and B) = P(A) $\times$ P(B | A).

Recall the hint: Say you have three numbers, a, b and c. Then if $a \times b = c$, then $b = c \div a$. That means we can rephrase Handy Rule #1:

Handy Rule #1 (rephrased):

> Say A and B are two "events" involving an experiment. Then
>
> P(B | A) = P(A and B) $\div$ P(A).

Therefore

P(I purple | II purple)

= P(I purple and II purple) $\div$ P(II purple)

So we need to find 2 probabilities:

- P(I purple and II purple)= $\frac{5}{15} \times \frac{5}{9} = \frac{5}{27}$.
- P(II purple) = $\frac{5}{15} \times \frac{5}{9} + \frac{10}{15} \times \frac{4}{9} = \frac{13}{27}$.

So,

P(I purple | II purple)

$= \frac{5}{27} \div \frac{13}{27}$

$= \frac{5}{27} \times \frac{27}{13}$

$= \frac{5}{13}$

≈ 0.38.

So, if you do this experiment in the next room, and you only tell me that the bean you got from II is purple, from my point of view the chances are about 38% that the one you got from jar I was purple.

In-class exercise:

Find P(I purple | II green). **Review the previous pages before you start!** Also, recall Handy Rule #4:

P(A) = 1 - P(not A)

Sometimes P(not A) is easier to find than P(A); then you just subtract.

Now, back to the original problem,

> P(the bean you get from II was originally in I | the bean you get from II is purple)

From the **rephrased** Handy Rule #1, we know that

P(bean from II was originally in I | bean from II is purple)

= P(bean from II was originally in I **and** bean from II is purple)

÷ P(bean from II is purple).

We already know that that second probability is $\frac{13}{27}$.

So, we need to find that first probability.

We will need to do a trick rephrasing:

P(bean from II was originally in I **and** bean from II is purple)

= P(bean from I was purple and bean from II was the one from I)

Now use (the original) Handy Rule #1:

P(bean from I was purple and bean from II was the one from I)

=

P(bean from I was purple)

$\times$ P(bean from II was the one from I | bean from I was purple)

= $\frac{5}{15} \times \frac{1}{9}$

= $\frac{1}{27}$.

So, at last we can solve the challenge problem:

P(bean from II was originally in I | bean from II is purple)

$= \frac{1}{27} \div \frac{13}{27}$

$= \frac{1}{27} \times \frac{27}{13}$

$= \frac{1}{13}$.