Dealing with “And”, “Or” in Probability Computations

Example:

> A certain pair of games, Game A and Game B, work as follows.
>
> You toss a die until get get a total of a least 4 dots from all of your tosses. If you are playing Game A, you win if your final total is exactly 5. If you are playing Game B, you win if your total is exactly 6.
>
> The rules say you must choose your game, A or B, <u>before</u> you start playing. Which one gives you a better chance of winning?

Kind of hard...let’s work on an easier problem first and come back to this one.

Example:

Suppose we have 2 jars, named Jar I and Jar II. Jar I contains 10 green jelly beans and 5 purple jelly beans, and Jar II contains 4 jelly beans of each color.

We draw out one jelly bean at random from Jar I, and then put it in Jar II. We then mix up Jar II and draw one jelly bean at random from Jar II.

We wish to find the following probabilities:

- P(bean from I is purple and bean from II is green)
- P(bean from II is green)
- P(at least 1 of the 2 beans drawn is green)

Keep in mind what those probabilities mean. Say we repeat the experiment many, many times, keeping a record in a notebook. Each line of the notebook records what happened in one repetition of the experiment. The notebook might look like this:

outcome	p from I, then g from II?	g from II?	at least 1 g?
p I, g II	yes	yes	yes
p I, g II	yes	yes	yes
g I, g II	no	yes	yes
p I, g II	yes	yes	yes
p I, p II	no	no	no
g I, p II	no	no	yes
p I, g II	yes	yes	yes
p I, p II	no	no	no
g I, p II	no	no	yes
...			
...			

Say we do this experiment 10,000 times. Then

- P(purple from I, green from II) is (approximately) the fraction of lines with “yes” in that second column
- P(green from II) is (approximately) the fraction of lines with “yes” in that third column
- P(at least 1 bean drawn is green) is (approximately) the fraction of lines with “yes” in that fourth column

We can also talk about **conditional** probabilities, such as

P(bean from II is green | bean from I is purple)

That vertical bar, |, is pronounced "given." We read

P(bean from II is green | bean from I is purple)

aloud as "the probability that the bean from Jar II is green, given that the bean from Jar I is purple." It means this:

> Say you have just drawn a bean from Jar I, and you saw that it is purple. You threw it into Jar II, and you are just about to draw a bean from Jar II. There are now 4 green and 5 purple beans in Jar II. So, your chances of now getting a green bean from Jar II are 4 out of 9. So,
> P(bean from II is green | bean from I is purple) = $\frac{4}{9}$.

Note that that means if you do the experiment, say, 10,000 times, then **among those notebook lines in which the bean from Jar I is purple, 4/9 of those lines** will have the bean from Jar II being green.

Now, how do we find those probabilities?

We could try "enumerating" (that means listing) all the possible outcomes of this experiment. But there are too many of them! And it would be worse with three jars, etc. (In some problems, there are even infinitely many possible outcomes.)

So, we need better ways.

First, we need

Handy Rule #1:

> Say A and B are two "events" involving an experiment.
> Then P(A and B) = P(A) × P(B | A).

So now we can find P(I purple and II green). Here

A = “the bean from I is purple”

B = “the bean from II is green”

$$P(I\ purple\ and\ II\ green) = P(I\ purple) \times P(II\ green | I\ purple)$$

So

$$P(I\ purple\ and\ II\ green) = \frac{5}{15} \times \frac{4}{9} = \frac{4}{27}.$$

Now, what about P(bean from II is green)?

Handy Rule #2:

> "Break big events into small events."
>
> That means to break the event you have into a bunch of "and" and "or" sub-events.

Here, that means, break

> "bean from II is green"

into

> "I purple and II green or I green and II green"

So

P(bean from II is green) =

P(I purple and II green or I green and II green)

Now, what do we find that?

Handy Rule #3:

If the events A and B don't overlap, then

P(A or B) = P(A) + P(B)

We can apply this to finding

P(I purple and II green or I green and II green)

Here

A = "I purple and II green"
B = "I green and II green"

(A and B do NOT overlap. A refers to all the notebook lines which say "I purple and II green", and B is all the lines which say "I green and II green". These are nonoverlapping lines.)

So,

P(bean from II is green) =

P(I purple and II green or I green and II green) =

P(I purple and II green) + P(I green and II green)

We already found P(I purple and II green) a couple of pages ago; it's 4/27.

In-class work:

Finish this yourself now, in groups. Find P(bean from II is green).

Homework:

Main problem:

Let's change the rules of the jelly bean problem a little: If the bean you draw from Jar I is green, you eat the jelly bean instead of throwing it in Jar II; if the jelly bean you draw from Jar I is purple, then you do throw it into Jar II.

Find:

- P(the jelly bean you draw from Jar II is green)
- P(at least one of the two beans drawn is green)

Challenge problem:

Review the rules of Games A and B on the first page. Calculate which game is better, and state the probability of winning that game.

Next week:

We will finally be able to solve the poker-hand problem.