Name: ______________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

**1.** (15) Consider the example in pp.163ff. Say we observe 8 of the logins to Jill's account. Assume they are independent and that they are really Jill. Let $M$ denote the number of logins in which Jill accesses at least 535 disk sectors. Find $Var(M)$.

**2.** (15) Suppose $X$ has a N(10,4) distribution. Give code that finds the number $c$ for which $P(X < c) = 0.168$.

**3.** Suppose $f_X(t) = ct^3$ on (0,4), 0 elsewhere.

(a) (10) Find $c$.

(b) (10) Find $EX$.

(c) (10) Find $E[\max(X, 1)]$.

(d) (15) Give a function **rt3(n)** that generates **n** random numbers from this density.

**4.** (15) On p.172, it states, "One can also show that Var(Y) = 2k." Deduce from this the value of $E(X^4)$, where $X$ has a N(0,1) distribution.

**5.** (10) Consider the museum example of the Central Limit Theorem in Sec. 8.15. We want to explain the CLT using a "notebook" view. Fill in the following blanks with terms or symbols from these sources: Theorem 14; Section 8.15; and "notebook-ology." (Example terms from the latter are *row*, *column*, *long-run proportion*, *repetition*, *experiment* and so on.) The number of rows of pins is __________________________, and the number of balls is __________________________,

**Solutions:**

**1.** $M$ is binomial, so $Var(Y) = np(1-p) = 8(0.01)(1-0.01)$.

**2.**

```
qnorm(0.168,10,2)
```

**3.a**

$$1 = \int_0^4 ct^3 \; dt, \tag{1}$$

so $c = 1/64$

**3.b**

$$EX = \int_0^4 t \cdot \frac{1}{64} t^3 \; dt \tag{2}$$

**3.c**

$$E[max(X,1)] = \int_0^1 1 \cdot \frac{1}{64} t^3 \; dt + \int_1^4 t \cdot \frac{1}{64} t^3 \; dt \tag{3}$$

**3.d** We need $F_x^{-1}(t)$.

$$t = F_X(s) = \frac{1}{64} \cdot \frac{1}{4} v^4 \Big|_0^s = \frac{1}{256} s^4 \tag{4}$$

Solve for $s$:

$$F_x^{-1(t)} = s = 4t^{0.25} \tag{5}$$

So the function is

```
rt3 <- function(n) 4 * (runif(n))^{0.25}
```

**4.** For convenience, let $W = X^2$. Then

$$E(X^4) = E[W^2] = Var(W) + (EW)^2 \tag{6}$$

But we are told in the text that $Var(Y) = 2k$. In (8.34), each $Z_i^2$ is a $W$, and they are independent. Thus we must have

$$Var(W) = 2 \tag{7}$$

Also, (8.36) shows that $EW = 1$. So (**??**) shows that

$$E(X^4) = 3 \tag{8}$$

**5.** $n$, number of rows of the notebook.