Name: _____

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (15) Suppose the random variable Y has density ct^3 on the interval (1,3), 0 elsewhere. What must be the value of the constant c?

2. (15) Consider the light bulb example, p.139 top. We keep sampling bulbs until we get one with lifetime less than 2.8. Let N denote the number of bulbs we sample. Find $Var(N)$.

3. Consider the Markov analysis of the bus ridership problem, pp.125-126.

(a) (10) Find p_{21} (the probability of going from state 2 to state 1).

(b) (15) Find the long-run proportion of stops at which there will be exactly one disappointed customer, among those cases in which no one alights. Write your answer in terms of the π vector, using R notation, e.g. `pi[12]^3`.

4. (15) Recall that the *skewness* of a random variable X is defined to be

$$E[(X-\mu)^3/\sigma^3] \tag{1}$$

Using R's **integrate()**, find the skewness of the random variable X in the bottom half of p.138.

5. (15) We found that the mean time between wins in the 3-consecutive-heads game is about 14.1. Express this mean using notation P(), E(), Var() and the T_{ij} in Section 6.10. (In your electronic file, write the latter quantity as T_ij.)

6. (15) Suppose we have parametric family of densities called the *nasty* family, with support $(0,\infty)$. So we should have functions **dnasty()**, **pnasty()** and so on, but have only been given **dnasty()**. Say the latter has parameters **x** (a scalar, not a vector) and **nastyparm**, the parameter indexing the family. The arguments for **pnasty()** will be **x** and **nastyparm**. Show R code that constructs **pnasty()** from **dnasty()**.

Solutions:

1. The density must integrate to 1.0, so $c = 1/20$.

2. N has a geometric distribution with $p = (2.8^2 - 1)/15$.

3.a We have 2 on the bus, and want the probability of this count dropping to 1. Either 2 alight and 1 boards, or 1 alights and 0 board. So the probability is $0.2^2 \cdot 0.4 + 2(0.2)(0.8) \cdot 0.5$.

3.b $\pi_{20}0.4 + \pi_{19}0.1$

4.

```
integrate(function(x) ((x-2.8)/sqrt(0.66))^3 * 2 * x / 15,1,4)
```

5. $E(T_{00})$

6.

```
function pnasty(x,nastyparm) integrate(dnasty,0,x,nastyparm)$value
```