Name: ______________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. This problem concerns the parking place model, Section 4.2.2.

(a) (15) Let X denote the number of empty spaces in the first block. Find $Var(X)$.

(b) (15) Find $p_D(2)$.

(c) (15) A group of friends in 3 cars needs 3 spaces. They will take the first 3 that they find. Let L denote the space number of the spot in which the third car ends up. (The first car will take the first space, and so on.) Find $P(L = 15)$.

2. (20) In (4.31), suppose n is 3 rather than 2. Find $F_X(1)$.

3. Consider (4.26). The mean and variance of X should be given by (4.29) and (4.30). But suppose due to a flaw in the recording process, there is a 0.20 chance that B_5 is a duplicate of B_4. Assume $0 < p < 1$.

(a) (20) State which one of the following will be true.

(i) $EX = np$

(ii) $EX < np$

(iii) $EX > np$

(iv) None of the above is necessarily true.

(b) (15) State which one of the following will be true.

(i) $Var(X) = np(1 - p)$

(ii) $Var(X) < np(1 - p)$

(iii) $Var(X) > np(1 - p)$

(iv) None of the above is necessarily true.

Solutions:

1.a The random variable in question is binomially distributed, so the variance is given by (4.30), $10(0.15)(1-0.15)$.

1.b

$$P(D=2) = P(N=9 \text{ or } N=13) = (1-0.15)^8 0.15 + (1-0.15)^{12} 0.15 \tag{1}$$

1.c This is negative binomial. Use (4.36), $k = 15,\ r = 3,\ p = 0.15$.

2.

$$F_X(1) = P(X \leq 1) = P(X = 0 \text{ or } X = 1) = (1-0.5)^3 + 3(1-0.5)^2 0.5 \tag{2}$$

3.a Answer (i) is correct. Eqn. (4.29) uses (3.19), which is true regardless of whether the terms are independent. The fact that the last two terms are not independent, indeed are equal, is irrelevant. The mean of B_5 will be p whether the flaw surfaces or not.

3.b Answer (iii) is correct. Eqn. (4.30) uses (3.75), which requires independence. Otherwise we must use (3.76), in which most of the covariance terms will be 0 but one will be positive.