Name: ______________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

**1.** (20) Your chemistry professor announces the results of the midterm exam: Mean 18 and standard deviation 5. She says that since the scores were so low, she is going to multiply all scores by 2 and then add 10. What will be the new mean and standard deviation? Answer using R's **c()** notation, e.g. **c(88,-8)** if you think the new mean and standard deviation will be 88 and -8, respectively.

**2.** (25) Consider the ALOHA example, Sec. 2.5 (and $p = 0.4$, $q = 0.8$). Let $O_k$ denote the number of original messages that are still pending at the end of epoch $k$, $k = 1, 2, ....$ We are just concerned with $k = 1$. Find $E(O_1)$.

**3.** (15) Suppose $K$ and $L$ are independent indicator random variables, with event probabilities $p$ and $q$. Supply the reasons for each step in the following derivation, in which $a$ and $b$ are constants. The reasons should cite equations or properties, maybe algebra, say with Eqns. (2.10)-(2.13) as an example. You will have answers (a), (b) and (c), i.e. 6 lines in your electronic file.

$$
\begin{aligned}
Var(aK + bL) &= Var(aK) + Var(bL) \quad \text{(reason (a))} \\
&= a^2 Var(K) + b^2 Var(L) \quad \text{(reason (b))} \\
&= a^2 p(1-p) + b^2 q(1-q) \quad \text{(reason (c))}
\end{aligned}
$$

**4.** (20) Say we have a random variable $X$, of which we simulate many instances, resulting in an R vector **w**. We make the R call

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mean(w > mean(w))
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State what quantity this is approximating. Your answer must use math symbols such as E(), P(), Var(), X and punctuation — no code and no English.

**5.** (20) In the context of p.65, find $Cov(G_1, G_2)$.

**Solutions:**

**1.**

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c(46,10)
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**2.**

$$1 \times 2(0.4)(1-0.4) + 2 \times (0.4^2 + (1-0.4)^2) \tag{1}$$

**3.a** Eqn. (3.75)

**3.b** Property G

**3.c** Eqn. (3.79)

**4.** $P(X > EX)$

**5.**

$$\begin{aligned} Cov(G_1, G_2) &= E(G_1 G_2) - E(G_1) \cdot E(G_2) && (2) \\ &= r - (\frac{2}{3})^2 && (3) \end{aligned}$$

where

$$r = P(G_1 G_2 = 1) = \frac{2}{3} \cdot \frac{5}{8} \tag{4}$$

Note that $G_1 G_2$ is itself an indicator random variable.