Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (25) Consider the bus ridership example, Section 2.12. State in which ones of the following pairs the events A and B are disjoint. Your answer might be, (i) and (v), for instance.

(i) A is $B_1 = 2$, B is $B_2 = 2$.

(ii) A is $B_1 = 0$, B is $L_2 = 1$.

(iii) A is $B_1 = 1$, B is $L_2 = 1$.

(iv) A is $B_1 = 0$, B is $B_2 + B_3 = 4$.

(v) A is $L_3 = 0$, B is two people alight at stop 4.

2. (25) Again consider the bus ridership example, Section 2.12. Find $P(B_1 = 0 \mid L_2 = 0)$.

3. (25) The function **alohasim()** below simulates **nreps** repetitions of the ALOHA model for **nepochs** epochs, **nnodes** nodes, with **p**, **q** as in the book. The return value is a matrix of **nepochs** rows and **nreps** columns, showing the number of active nodes at the end of each epoch. (25) Fill in the blanks.

```
alohasim <-
   function(nreps,nepochs,nnodes,p,q) {
   replicate(nreps,
      simepochs(nepochs,nnodes,p,q))
}

simepochs <- function(nepochs,nnodes,p,q) {
   # make space for results
   nactivevals <- vector(length=nepochs)
   # initial condtion
   nactive <- nnodes
   for (i in 1:  blank (a)  ) {
      if (nactive < nnodes) {
         ninactive <- nnodes - nactive
         nactive <-
            nactive + nheads(ninactive,
               blank (b) )
      }
      ntrysend <-  blank (c)
      if (ntrysend ==  blank (d)  )
         nactive <- nactive - 1
      nactivevals[i] <- nactive
   }
   # R auto returns last value computed
   nactivevals
}

# simulate tossing nc coins, each with
# probability r of heads; return number
# of heads
nheads <- function(nc,r) {
    tmp <- runif(nc)
    sum(  blank (e)  )
}
```

4. (25) (Continuation of Problem 3.) State, using symbols like P(), X5 and so on, what approximate probability the following is finding.

```
z <- alohasim(5000,2,2,0.4,0.8)
w <- which(z[1, ] == 1)
z1 <- z[ ,w]
mean(z1[2,] == 2)
```

Solutions:

1. (v)

2.

$$P(B_1 = 0 \mid L_2 = 0) = \frac{P(B_1 = 0, L_2 = 0)}{P(L_2 = 0)} \tag{1}$$

The denominator was already computed in the textbook, as 0.292. The numerator is

$$P(B_1 = 0)\ P(L_2 = 0 \mid B_1 = 0) = 0.5^2 \tag{2}$$

So the answer is $0.5^2/0.292$.

3.a

```
alohasim <- function(nreps,nepochs,nnodes,p,q) {
   replicate(nreps,simepochs(nepochs,nnodes,p,q))
}

# simulates and records nepochs epochs; return value has the number of
# active nodes at the end of each epoch
simepochs <- function(nepochs,nnodes,p,q) {
   # make space for results
   nactivevals <- vector(length=nepochs)
   # initial condtion
   nactive <- nnodes
   # simulate the epochs
   for (i in 1:nepochs) {
      if (nactive < nnodes) {
         ninactive <- nnodes - nactive
         nactive <- nactive + nheads(ninactive,q)
      }
      ntrysend <- nheads(nactive,p)
      if (ntrysend == 1)
         nactive <- nactive - 1
      nactivevals[i] <- nactive
   }
   nactivevals  # R auto returns last value computed
}

# simulate tossing nc coins, each with probability r of heads; return
# number of heads
nheads <- function(nc,r) {
    tmp <- runif(nc)
    sum(tmp < r)
}
```

3.b $P(X_2 = 2 \mid X_1 = 1)$