Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (15) On Section 6.4, find $E(X^3)$.

2. (15) On p.119, suppose Z is the number of heads obtained from three tosses of a coin, rather than two. Find $F_Z(1.88)$. Write your answer only as a numerical expression, NO calls to R functions.

3. (15) Suppose $f_X(t) = ct^2$ for $0 < t < 2$, 0 elsewhere, for some constant c. Find c.

4. (15) Consider the coin-and-die game, Section 4.15.3. You don't observe the game personally, but you hear that the player took at most 2 turns to roll a 5. Find the probability that the player wins exactly \$1.

5. (15) The following simulation finds and returns the long-run average seek distance in the disk drive model, pp.126ff. Fill in the blanks:

```
sim <- function(nreps) {
   # start at the middle track,
   # but doesn't matter
   oldtracknum <- 0.5
   seeks <- vector(length=nreps)
   for (i in 1:nreps) {
      tracknum <-  blank (a)
      seeks[i] <-  blank (b)
      oldtracknum <- tracknum
   }
   blank (c)
```

6. Consider the Markov inventory model, p.112, and the following run of the code:

```
> inventory(0.8,0.2,5)
[1] 0.1936083 0.1932367 0.1950948
0.1858045 0.2322557
```

(a) (15) Find the proportion of days in which a customer leaves emptyhanded.

(b) (10) Find the proportion of customers who leave emptyhanded.

Solutions:

1.

$$\int_1^4 t^3 \cdot 2t/15 \; dt$$

2.

```
pbinom(1,3,0.5)
```

3. The density must integrate to 1. Solving for c yields the value 3/8.

4.

$$P(W=1|M \leq 2) = \frac{P(W=1 \text{ and } M \leq 2)}{P(M \leq 2)}$$

The denominator is

$$\frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6}$$

and the numerator is

$$P(W=1 \text{ and } M=1) + P(W=1 \text{ and } M=2) = \frac{1}{6} \cdot \frac{1}{2} + \frac{5}{6} \cdot \frac{1}{6} \cdot 2(1-\frac{1}{2})\frac{1}{2}$$

5.

```
sim <- function(nreps) {
   oldtracknum <- 0.5
   seeks <- vector(length=nreps)
   for (i in 1:nreps) {
      tracknum <- runif(1)
      seeks[i] <- abs(tracknum - oldtracknum)
      oldtracknum <- tracknum
   }
   mean(seeks)
}
```

6.a

$$0.1936083 \cdot 0.2$$

6.b Think of what will happen over the course of 10000 days. We will have approximately 12000 customers, among whom

$$0.1936083 \cdot 0.2 \cdot 10000$$

will leave emptyhanded. Then divide.