Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (10) In the phrasing of the article on Microsoft's new Azure software, the firm aims the app to ________ machine learning.

2. (10) A trucking company transports many things, including furniture. Let X be the proportion of a truckload that consists of furniture. For instance, if 15% of given truckload is furniture, then X = 0.15. We have data on X, and will plot histograms and so on, in order to find a good model. Suggest a good parametric distribution family for modeling X.

3. This problem concerns the material in Section 5.5.5, The random variables T_8 and Y are as in the bottom of p.122 and the top of p.124, respectively. In the case of T_8, say the mean lifetime of ligh bulbs is 120 hours. If you need the $\Gamma()$ function, R offers it as **gamma()**. Also, there are **factorial()** and **exp()**.

(a) (10) State the value of $Var(Y)$.

(b) (10) Find $f_Y(88)$.

(c) (15) Find $E(1/Y)$.

(d) (10) Concerning T_8, we've asked someone to notify us when the eighth bulb burns out. At time 102.2 hours after the first bulb is installed, we still haven't heard from our notifier. Find the probability that at time 222.1, we still have not been notified.

(e) (15) The text remarks that in Figure 5.2, the curve for r = 10.0 is already looking rather bell-shaped. By calling **pnorm()**, we can find the normal approximation to, say, the cdf corresponding to that density, evaluated at 14.2. What would be our third argument in that function? Say that figure used $\lambda = 1$.

4. (10) Suppose $f_W(t) = t^{-2}$ for $t > 1$, 0 otherwise. Find the median of W, i.e. the 0.5 quantile.

5. (10) Suppose Z_1 and Z_2 are independent, each having distribution N(0,1). Find $Var(Z_1^2 - Z_2^2)$.

Solutions:

1. democratize

2. beta distributions

3a. The gamma family has variance r/λ^2, so

$$Var(Y) = 5/0.01^2 \tag{1}$$

3b.

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dgamma(88,5,0.01
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3c. Use Property E:

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integrate(function(t) (1/t) * 1/(factorial(4)) * 0.01^5 * t^4 * exp(-0.01*t),0,Inf)
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3d.

$$P(T_8 > 222.1 \mid T_8 > 102.2) = P(T_8 > 222.1) \ / \ P(T_8 > 102.2) \tag{2}$$

Then use

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(1 - pnorm(222.1,5,0.01)) / (1 - pnorm(102.2,5,0.01))
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3e.

$$\text{variance } = r/\lambda^2 = 10 \tag{3}$$

So, we would use the standard deviation, $\sqrt{10}$.

4. We have $F_W(t) = 1 - 1/t$, so set $0.5 = 1/t$, yielding t = 2.

5.

$$Var(Z_1^2 - Z_2^2) = 2Var(Z_1^2) \tag{4}$$

Z_1^2 has a chi-square distribution with k = 1 degree of freedom. So, its variance is $2k = 2$. The original expression has value 4.