Name: ____________________________

Directions: MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. Consider Equation (3.64).

(a) (15) List (on one line), the equation number(s) of the mailing tubes used to justify the equality $Var(7 + 2I) = 4Var(I)$.

(b) (15) Give the equation number of the relation that justifies $4Var(I) = 4 \cdot 0.5(1 - 0.5)$.

2. (15) Give the number of the mailing tube that justifies (3.80).

3. Consider the variables G_i, p.56.

(a) (10) Find $P(G_2 = 1 \mid G_1 = 1)$.

(b) (15) Find $P(G_1 = G_2)$.

4. (15) Suppose X and Y are independent random variables, with $EX = 1$, $EY = 2$, $Var(X) = 3$ and $Var(Y) = 4$. Find $Var(XY)$.

5. (15) In a certain game, Person A spins a spinner and wins S dollars, with mean 10 and variance 5. Person B flips a coin. If it comes up heads, Person A must give B whatever A won, but if it comes up tails, B wins nothing. Let T denote the amount B wins. Find $Var(T)$.

Solutions:

1.a (3.47), (3.40)

2. (3.32)

3.a Given the first draw resulted in a man, there will be 5 men and 3 women left, so the probability is 5/8.

3.b The requested probability is that of getting two men or two women, (6/9) (5/8) + (3/9)(2/8).

4. Use the relations $E(UV) = EU \cdot EV$ (for independent U,V) and then use $Var(U) = E(U^2) - (EU)^2$ repeatedly:

$$\begin{aligned}
Var(XY) &= E(X^2Y^2) - [E(XY)]^2 && (1)\\
&= E(X^2) \cdot E(Y^2) - (EX \cdot EY)^2 && (2)\\
&= [Var(X) + (EX)^2] \cdot [Var(Y) + (EY)^2] - (EX \cdot EY)^2 && (3)\\
&= (3 + 1^2)(4 + 2^2) - (1 \cdot 2)^2 && (4)
\end{aligned}$$

5. Use (**??**), in this case with X = I, where I is an indicator variable for the event that B gets a head, and Y = S. Then T = IS, and I and S are independent, so

$$Var(T) = Var(IS) = [Var(I) + (EI)^2] \cdot [Var(S) + (ES)^2] - (EI \cdot ES)^2 \quad (5)$$

Then use the facts that I has mean 0.5 and variance 0.5(1-0.5), with S having the mean and variance given in the problem.