Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**, **pnorm()**, etc.

Note too the R function **integrate()**, e.g.

```
> integrate(function(x) x^2,0,1)
0.3333333 with absolute error < 3.7e-15
```

The limits of integration must be numbers or Inf or -Inf, not symbols. Thus one cannot use it for the inner integral in a double integral.

1. Consider the computer worm example in Sec. 8.3. Say g = 5.

(a) (25) Find the probability that the time spent at state 2 (before going to state 3) is greater than 0.10.

(b) (15) Let T_{35} denote the time needed to go from state 3 to state 5. Find $f_{T_{35}}(0.3)$.

2. In each of the following, state—using mathematical symbols, e.g. E(), P(), F, f, >, names of the variables used in the example (don't make up your own names), etc.—what the integral is calculating. Use an underscore for subscripts in your **Answers** file, e.g. write U_1 for U_1.

(a) (0) $\int_{0.2}^{0.8} 2(1-v)\, dv$ in Section 8.3.4.

(b) (25) $\int_0^1 \int_0^1 st(2-s-t)\, dt\, ds$ in Section 8.2.4.

(c) (20) $\int_1^8 \frac{1}{(8-s)^2} \frac{1}{s^2}\, ds$, where we have independent random variables X and Y with density $1/w^2$ on $(1, \infty)$.

3. (15) Suppose X and Y be independent and each have a uniform distribution on the interval (0,1). Let Z = min(X,Y). Find $f_Z(0.8)$.

Solutions:

1.a The text says that the time spent at state 2 is exponentially distributed with parameter 2x3 = 6. So, the probability is

```
1−pexpon(0.10,6}
```

1.b Have sum of two independent exponentials, but with different λ values—just like the backup battery example. So, $f_{T_{35}}(0.3)$ is equal to $\exp(-0.3/6) + \exp(-0.3/4$ (where $6 = 3 \cdot 2$ and $4 = 4 \cdot 1$).

2.a $P(0.2 < S < 0.8)$ (error in original problem statement)

2.b E(XY)

2.c $f_{X+Y}(8)$

3. Using the same derivation as on p.168,

$$
\begin{aligned}
F_Z(t) &= P(Z \leq t) && (1)\\
&= 1 - P(Z > t) && (2)\\
&= 1 - P(X > t \text{ and } Y > t) && (3)\\
&= 1 - (1-t)^2 && (4)
\end{aligned}
$$

So, $f_Z(t) = 2(1-t)$ and $f_Z(0.8) = 0.4$.

4. This is the density of X+Y, where X and Y are independent with the given density.