Name: _____

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SEND-ING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R func-tion, e.g. **choose()**, **pnorm()**, etc.

Note too the R function **integrate()**, e.g.

```
> integrate(function(x) x^2,0,1)
0.3333333 with absolute error < 3.7e−15
```

The limits of integration must be numbers or Inf or -Inf, not symbols. Thus one cannot use it for the inner integral in a double integral.

1. Say X and Y have means 1 and 2, with variances 4 and 8, and with covariance -1. Find the following:

(a) (20) Var(X+Y)

(b) (20) $\rho(X + Y)$

(c) (15) Cov(X,X+Y)

2. Find the cdf values:

(a) (15) In the marbles problem, pp.156-158, find $F_{Y,B}(1,1)$.

(b) (20) In the example in Sec. 8.2.3, find $F_{X,Y}(0.5, 0.2)$.

3. (10) In Sec. 7.3.5, find Var(X - 2Y + Z), using matrix methods. Note: Recall that in R, matrix multiplication is done via %*%, and that matrix storage is in column-major order, e.g.

```
1  > m <- matrix(c(5,12,13,3,4,5),ncol=2)
2  > m
3       [,1] [,2]
4  [1,]    5    3
5  [2,]   12    4
6  [3,]   13    5
```

Solutions:

1.a

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y) = 4 + 8 - 2 = 10 \tag{1}$$

1.b

$$\frac{-1}{\sqrt{4}\sqrt{8}} \tag{2}$$

1.c

$$Cov(X, X + Y) = Cov(X, X) + Cov(X, Y) = 4 - 1 = 3. \tag{3}$$

2.a $0.002 + 0.024 + 0.162 + 0.073$

2.b

$$\int_0^{0.2} \int_0^s 8st \ dt \ ds + \int_{0.2}^{0.5} \int_0^{0.2} 8st \ dt \ ds \tag{4}$$

3.

```
1   50 * c(1,-2,1) %*% matrix(c(5/36,-1/1,-1/12,-1/18,2/9,-1/6,-1/12,-1/6,1/4) %*% c(1,-2,1)
```