Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**.

1. Consider the random variable X on p.92.

(a) (15) Find the probability that X is between 0.1 and 0.2. [Should have been 1.1 and 1.2. Otherwise the probability is 0.]

(b) (15) Find $E(\sqrt{X})$.

2. Consider the ALOHA Markov chain example, beginning on p.68, but with 4 nodes in the network, not just 2.

(a) (10) How many rows will the P matrix now have?

(b) (15) Find p_{43}, for the case q = 0.2, p = 0.6.

3. Suppose light bulb lifetimes are exponentially distributed with mean 10.0 months. We try them one at a time, until we find the third one that lasts longer than 5.0. Let N denote the number of light bulbs we try.

(a) (15) What famous parametric family does the distribution of N belong to?

(b) (15) Find Var(N).

4. (15) In the network intrusion example, p.97, suppose Jill logs in twice. Let X and Y denote the number of disk sectors she reads in the two sessions, assumed to be independent. Find $P(X + Y > 1088)$.

Solutions:

1.a See note in problem statement. Probability is 0 as stated. For 1.1, 1.2:

$$P(1.1 < X < 1.2) = \int_{1.1}^{1.2} 2t/15 \; dt \tag{1}$$

1.b

$$E(\sqrt{X} = \int_{1}^{4} t^{0.5} 2t/15 \; dt = \frac{4}{75} \cdot 31 \tag{2}$$

2.a 5

2.b

$$p_{43} = \binom{4}{1} p^1 (1-p)^3 = 0.1536 \tag{3}$$

3.a Negative binomial.

3.b From (3.114):

$$Var(N) = r \cdot \frac{1-p}{p^2} \tag{4}$$

Here r = 3 and

$$p = \int_{5.0}^{\infty} 0.1 e^{-0.1t} \; dt \tag{5}$$

The integral can be computed by hand, or as

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1 - pexp(5.0,0.1)
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4. X+Y has a normal distribution with mean $2 \cdot 500$ and variance $2 \cdot 15^2$. The specified probability is then computed as

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1 - pnorm(1088,1000,sqrt(450))
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