Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**.

1. This problem concerns the bus ridership example, which begins in Sec. 2.11.

(a) (20) The bus driver has the habit of exclaiming, "What? No new passengers?!" every time he comes to a stop at which $B_i = 0$. Let N denote the number of the stop (1,2,...) at which this first occurs. Find P(N = 3).

(b) (20) Find Var(N) in (a).

(c) (20) Let T denote the number of stops, out of the first 6, at which 2 new passengers board. For example, T would be 3 if $B_1 = 2$, $B_2 = 2$, $B_3 = 0$, $B_4 = 1$, $B_5 = 0$, and $B_6 = 2$. Find $p_T(4)$.

(d) (20) The bus ridership problem is simulated in Section 2.12.4. Give a single call to a built-in R function that can replace lines 8-11.

2. (20) A machine contains one active battery and two spares. Each battery has a 0.1 chance of failure each month. Let L denote the lifetime of the machine, i.e. the time in months until the third battery failure. Find P(L = 12).

Solutions:

1.a N has a geometric distribution, with p = probability of 0 new passengers at a stop = 0.5. Thus $p_N(3) = (1-0.5)^2 0.5$, by (3.75).

1.b $Var(N) = (1-0.5)/0.5^2$, by (3.84).

1.c T has a binomial distribution, with n = 6 and p = probability of 2 new passengers at a stop = 0.1. Then

$$p_T(4) = \binom{6}{4} 0.1^4 (1-0.1)^{6-4} \tag{1}$$

Note that your electronic answer could be

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dbinom(4,6,0.1)
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1.d Given that k passengers are currently on the bus, the number who alight has a binomial distribution, with n = k and p = 0.2. The code in lines 8-11 is merely simulating the k trials, in this case with k being the variable named **passengers**. Thus the code could be replaced by

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passengers <- passengers - rbinom(1,passengers,0.2)
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2. The number of months until the third failure has a negative binomial distribution, with r = 3 and p = 0.1. Thus the answer is obtained by (3.111), with k = 12:

$$P(L = 12) = \binom{11}{2} (1-0.1)^9 0.1^3 \tag{2}$$