Name: _____

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

1. (20) The formal term used when two events cannot occur "in the same notebook line" is that they are _____.

2. Consider the simple board game in Section 2.10. Let X denote your position after your first turn.

(a) (20) Find P(X = 1).

(b) (20) Find P(R = 1 | X = 1).

3. Consider the ALOHA example, same as in the book, except that both nodes start out inactive, i.e. $X_0 = 0$. Assume p = 0.6, q = 0.2.

(a) (20) Find the probability that there is a collision in first epoch.

(b) (20) Find $P(X_1 = 2)$.

Solutions:

1. disjoint

2.a

$$P(X = 1) = P(R = 1 \text{ or } R = 3 \text{ and } B = 5) = \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}$$

2.b

$$
\begin{aligned}
P(R = 1 | X = 1) &= \frac{P(R = 1 \text{ and } X = 1)}{P(X = 1)} & (1) \\
&= \frac{P(R = 1)}{P(X = 1)} & (2) \\
&= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}} & (3)
\end{aligned}
$$

3.a How can it happen? A collision will occur in the first epoch if and only both nodes develop messages and both try to send, which has probability

$$q^2 p^2 = (0.2)^2 (0.6)^2$$

3.b How can it happen? $X_1 = 2$ will occur if and only if both nodes go active and either both send or both refrain from sending. the probability of that is

$$q^2 [p^2 + (1 - p)^2] = (0.2)^2 [0.6^2 + 0.4^2]$$