Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code or a numerical answer, you are allowed to use any built-in R function, e.g. **choose()**, **pnorm()**, etc.

1. Suppose X has the density 2(1-t) for t in (0,1) and 0 elsewhere.

(a) (10) Find $P(X > 0.5)$.

(b) (10) Find EX.

(c) (15) Find $F_X(0.2)$.

(d) (10) Could the density be 5(1-t) on (0,1)? Explain why or why not (*in one line*!).

2. Sam has a die that he wants to test for fairness, in this case defined to be $H_0 : p = 1/6$, where p is the probability of getting a 1. He will roll the die until he gets the first 1; let N_1 denote the number of rolls needed. He'll then do it again; let N_2 denote the number of rolls needed this time. He'll do it n times in all. Then he will calculate

$$\overline{N} = \frac{N_1 + ... + N_n}{n} \tag{1}$$

Suppose, unknown to Sam, the actual value of p is 0.20.

(a) (10) Find $P(N_1 = 5)$.

(b) (15) Find (exactly) $P(\overline{N} = 1.5)$ for the case n = 2. (Remember to ask yourself, "How can it happen?")

(c) (10) Find (approximately) $P(\overline{N} < 5.2)$ for the case n = 100.

(d) (10) What will be the approximate value of the quantity $\frac{1}{100} \sum_{i=1}^{100} N_i^2$?

(e) (10) Again assuming n = 100, suppose Sam finds that $\overline{N} = 7.5$. Find the p-value.

Solutions:

1.a

$$P(X > 0.5) = \int_{0.5}^{1.0} 2(1-t)\ dt = 1/4$$

1.b

$$EX = \int_{0.5}^{1.0} t \cdot 2(1-t)dt = 1/3$$

1.c

$$P(X < 0.2) = \int_{0}^{0.2} 2(1-t)\ dt = 0.36$$

1.d No, integral would be > 1.

2.a

$$0.8^4 \cdot 0.2$$

2.b

$$P(N_1 = 1 \text{ and } N_2 = 2 \text{ or } N_2 = 1 \text{ and } N_1 = 2) = 2 \cdot 0.2 \cdot 0.8 \cdot 0.2$$

2.c $\overline{N}$ is approximately normally distributed with mean $1/0.2 = 5$ and variance $(0.8/(0.2)^2)/100$. Thus the probability is

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pnorm(5.2,mean=5.0,sd=sqrt(0.8/(0.2^2)/100))
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2.d Use (9.21). The quantity is question is

$$s^2 + \overline{N}^2$$

which for large n is approximately

$$Var(N) + (EN)^2 = 0.8/(0.2^2) + 5^2 = 45.$$

2.e Using similar reasoning as in **2.c**:

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2*(1 - pnorm(7.5,mean=6.0,sd=sqrt((0.83/(0.17^2)/100))))
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