Name: _____

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

Unless otherwise stated, give numerical answers as expressions, e.g. $\frac{2}{3} \times 6 - 1.8$. Do NOT use calculators.

1. Suppose the random vector $X = (X_1, X_2, X_3)'$ has mean $(2.0, 3.0, 8.2)'$ and covariance matrix

$$\begin{pmatrix} 1 & 0.4 & -0.2 \\ & 1 & 0.25 \\ & & 3 \end{pmatrix} \quad (1)$$

(a) (10) Fill in the three missing entries.

(b) (10) Find $Cov(X_1, X_3)$.

(c) (10) Find $\rho(X_2, X_3)$.

(d) (10) Find $Var(X_3)$.

(e) (15) Find the covariance matrix of $(X_1 + X_2, X_2 + X_3)'$.

(f) (15) If in addition we know that X_1 has a normal distribution, find $P(1 < X_1 < 2.5)$, in terms of $\Phi()$.

(g) (15) Consider the random variable $W = X_1 + X_2$. Which of the following is true? (i) $Var(W) = Var(X_1 + X_2)$. (ii) $Var(W) > Var(X_1 + X_2)$. (iii) $Var(W) < Var(X_1 + X_2)$. (iv) In order to determine which of the two variances is the larger one, we would need to know whether the variables X_i have a multivariate normal distribution. (v) $Var(X_1 + X_2)$ doesn't exist.

2. (15) What is the (approximate) output of this R code:

```
count <- 0
for (i in 1:10000) {
   count1 <- 0
   count2 <- 0
   count3 <- 0
   for (j in 1:20) {
      x <- runif(1)
      if (x < 0.2) {
         count1 <- count1 + 1
      } else if (x < 0.6) count2 <- count2 + 1 else
            count3 <- count3 + 1
   }
   if (count1 == 9 && count2 == 2 && count3 == 9) count <- count + 1
}
cat(count/10000)
```

Solutions:

1a.

$$\begin{pmatrix} 1 & 0.4 & -0.2 \\ 0.4 & 1 & 0.25 \\ -0.2 & 0.25 & 3 \end{pmatrix} \quad (2)$$

1b. -0.2

1c. $\frac{0.25}{\sqrt{1}\sqrt{3}}$

1d. 3

1e.

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.4 & -0.2 \\ 0.4 & 1 & 0.25 \\ -0.2 & 0.25 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

1f. $\Phi(\frac{2.5-2.0}{1}) - \Phi(\frac{1-2.0}{1})$

1g. (ii), by (3.29)

2.

$\frac{20!}{9!^2 2!} 0.2^9 0.4^{11}$