Name: ----------

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

Unless otherwise stated, give numerical answers as expressions, e.g. $\frac{2}{3} \times 6 - 1.8$. Do NOT use calculators.

1. (15) Consider random variables X_1 and X_2, for which $Var(X_i) = 1.0$ for i = 1,2, and $Cov(X_1, X_2) = 0.5$. Find $Var(X_1 + X_2)$.

2. (15) Suppose we have random variables X and Y, and define the new random variable Z = 8Y. Then which of the following is correct? (i) $\rho(X, Z) = \rho(X, Y)$. (ii) $\rho(X, Z) = 0$. (iii) $\rho(Y, Z) = 0$. (iv) $\rho(X, Z) = 8\rho(X, Y)$. (v) $\rho(X, Z) = \frac{1}{8}\rho(X, Y)$. (vi) There is no special relationship.

3. Suppose $f_X(t) = 2t$ for $0 < t < 1$ and the density is 0 elsewhere.

(a) (10) Find $h_X(0.5)$.

(b) (10) Which statement concerning this distribution is correct? (i) IFR. (ii) DFR. (iii) U-shaped failure rate. (iv) Sinusoidal failure rate. (v) Failure rate is undefined for $t > 0.5$.

4. (15) Consider the coin game on p.33. Find $F_{X_3,Y_3}(0, 0)$.

5. (15) In the backup battery example on p.85, find Var(W).

6. (10) Consider the "8st" density example on p.74. Find $P(Y > X^2)$. Express your answers as a definite integral, ready for any calculus student to compute an actual number from.

7. (10) What will be the (approximate) output of the following R code?

```
s <- 0
s2 <- 0
for (rep in 1:10000) {
   z3 <- rnorm(3)  # generate 3 N(0,1) random variates
   tot <- sum(z3^2)  # sum of the squares of the 3 variates
   s <- s + tot
   s2 <- s2 + tot^2
}
m <- s/10000
print(m)
print(s2/10000 - m^2)
```

Solutions:

1. 3

2. (i)

3a. $h_X(t) = 2t/(1 - t^2)$, so $h_X(0.5) = 4/3$

3b. IFR

4 $F_{X_3,Y_3}(0, 0) = P(X_3 \leq 0 \text{ and } Y_3 \leq 0) = P(X_3 = 0 \text{ and } Y_3 = 0) = 0.5^3 \cdot 0.5$

5. $Var(W) = Var(X+Y) = Var(X)+Var(Y) = 2^2+1^2$

6.

$$\int_0^1 \int_{s^2}^s 8st \; dt \; ds$$

7. Using Sections 2.3.3.1 and 2.3.5.1, and (1.4.6), we have that the output will be 3 and 6.