Name: _____

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

Unless otherwise stated, give numerical answers as expressions, e.g. $\frac{2}{3} \times 6 - 1.8$. Do NOT use calculators.

1. (20) Fill in the blank: Density functions for continuous random variables are analogs of the _____ functions that are used for discrete random variables.

2. (20) Suppose for some random variable W, $F_W(t) = t^3$ for $0 < t < 1$, with $F_W(t)$ be 0 and 1 for $t < 0$ and $t > 0$, respectively. Find $f_W(t)$ for $0 < t < 1$.

3. (10) Suppose X has a binomial distribution with parameters n and p. Then X is approximately normally distributed with mean np and variance np(1-p). For each of the following, answer either A or E, for "approximately" or "exact," respectively:

(a) (10) distribution of X is normal

(b) (10) E(X) is np

(c) (5) Var(X) is np(1-p)

4. Suppose light bulb lifetimes have an exponential distribution with mean 100.0 hours, i.e. $\lambda = 0.01$. We use our first light bulb, with it lasting for X_1 hours. When it burns out, we replace it with a second bulb, which lasts X_2 hours. Then $T_2 = X_1 + X_2$ is the time of the second replacement.

(a) (10) Give numerical expressions for the mean and variance of T_2.

(b) (5) State $f_{T_2}(t)$ (the actual function, not the name of a family etc.).

(c) (10) Fill in the blank:

$$P(T_2 > t) = P[N(t)\_\_\_\_\_]$$

Solutions:

1. probability mass functions

2. $3t^2$

3a. A (pages 30-31)

3b. E (pages 30-31)

3c. E (pages 30-31)

4a. $2 \cdot 100$, $2 \cdot 100^2$ (page 59)

4b. $0.01^2 t e^{-\lambda t}$ (page 59)

4c. ≤ 1 (like (2.38))