Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

Unless otherwise stated, give numerical answers as expressions, e.g. $\frac{2}{3} \times 6 - 1.8$. Do NOT use calculators.

1. (15) Using Equation (1.64), give a numerical expression for $Var(X_1)$.

2. Suppose X and Y are independent random variables with standard deviations 3 and 4, respectively.

(a) (10) Find Var(X+Y).

(b) (10) Find Var(2X+Y).

3. (30) Fill in the blanks in the following simulation, which finds the approximate variance of N, the number of rolls of a die needed to get the face having just one dot.

```
onesixth <- 1/6
sumn <- 0
sumn2 <- 0
for (i in 1:10000) {
   n <- 0
   while(TRUE) {
      ________________________________________
      if (______________________________ < onesixth) break
   }
   sumn <- sumn + n
   sumn2 <- sumn2 + n^2
}
approxvarn <- ____________________________________________
cat("the approx. value of Var(N) is ",approx,"\n")
```

4. (20) Jack and Jill keep rolling a four-sided and three-sided die. The first player to get the face having just one dot wins, except that if they both get a 1, it's a tie, and play continues. Let N denote the number of turns needed. Find $p_N(1)$.

5. (15) Let X be the total number of dots we get if we roll three dice. Find an upper bound for $P(X \geq 15)$, using our course materials.

Solutions:

1. $Var(X_1) = E(X_1)^2 - (EX_1)^2$. The last term is 1.52^2, and the next-to-last is $1^2 \cdot 0.48 + 2^2 \cdot 0.52$.

2. By (1.61), $Var(X+Y) = Var(X) + Var(Y) = 3^2 + 4^2$. By (1.48), $Var(2X) = 2^2 Var(X) = 2^2 \cdot 3^2$, so again by (1.61), $Var(2X+Y) = 2^2 \cdot 3^2 + 4^2$.

3.

```
n <- n + 1
runif(1)
sumn2/10000 - (sumn/10000)^2
```

4.

$$
\begin{aligned}
p_N(1) &= P(N = 1) \\
&= P(\text{Jack gets 1 and Jill doesn't or vice versa}) \\
&= \frac{1}{4} \cdot \frac{2}{3} + \frac{3}{4} \cdot \frac{1}{3}
\end{aligned}
$$

5. Use Markov's Inequality:

$$
P(X \geq 15) \leq \frac{EX}{15} = \frac{3(3.5)}{15}
$$

(Of course, it's a very poor bound in this case.)