

Name: \_\_\_\_\_

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

**Unless otherwise stated, give numerical answers as expressions, e.g.  $\frac{2}{3} \times 6 - 1.8$ . Do NOT use calculators.**

1. (20) Consider the ALOHA example from the text, for general  $p$  and  $q$ , and suppose that  $X_0 = 0$ , i.e. there are no active nodes at the beginning of our observation period. Find  $P(X_1 = 0)$ .

2. Consider a three-sided die, as opposed to the standard six-sided type. The game is to keep rolling the die until we get a total of at least 3. Let  $N$  denote the number of times we roll the die. For example, if we get a 3 on the first roll,  $N = 1$ . If we get a 2 on the first roll, then  $N$  will be 2 no matter what we get the second time. The largest  $N$  can be is 3. The rule is that one wins if one's final total is exactly 3.

(a) (20) Find the probability of winning.

(b) (20) Find  $P(\text{our first roll was a 1} \mid \text{we won})$ .

(c) Extra Credit: How could we construct such a die?

3. Consider the ALOHA simulation example on pp.11-12.

(a) (20) Suppose we wish to find  $P(X_2 = 1 \mid X_1 = 1)$  instead of  $P(X_2 = 2 \mid X_1 = 1)$ . What line(s) would we change, and how would we change them?

(b) (20) In which line(s) are we in essence checking for a collision?

**Solutions:**

1.  $(1 - q)^2 + 2q(1 - q)p$

2a.  $P(\text{win}) = P(3 \text{ or } 1, 2 \text{ or } 2, 1 \text{ or } 1, 1, 1) = \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3$

2b.  $P(\text{1st was 1} \mid \text{we won}) = \frac{P(\text{1st was 1 and we won})}{P(\text{we won})}$

$P(\text{1st was 1 and we won}) = P(1, 2) = \left(\frac{1}{3}\right)^2$

Then divide.

2c. For example, construct the die as a cylinder, with the proper ratio of height to radius to achieve the right balance.

3a. Line 34, writing `X2 == 1`, and making the same change in the output labeling in line 40. (Latter not counted wrong if missing.)

3b. Line 13.