Name: ______________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

Unless otherwise stated, give numerical answers as expressions, e.g. $\frac{2}{3} \times 6 - 1.8$. Do NOT use calculators.

1. A vending machine stocks Kit Kats, Butterfingers and Crunch bars. Among all customers, $\frac{1}{2}$ choose Kit Kats, $\frac{1}{3}$ select Butterfingers and $\frac{1}{6}$ purchase Crunches. Let K, B and C be the number of each item bought, among n transactions.

(a) (10) Suppose n = 5. To what parametric family does the distribution of the vector (K,B,C) belong, and with what parameter values?

(b) (10) Same question as (a), but for K alone.

(c) (10) For n = 6, give the value of Cov(K,C).

(d) (10) Suppose the stock on hand of Crunches is 3. (We have ther two items.) Let N be the number of customers we must observe before the Crunches are gone. To what parametric family of oes p_N belong, and with what parameter values??

2. (15) The R code below simulates the "museum demonstration" (pp.55-56). Fill in the blanks.

```
nrows <- 15
nballs <- 500
bins <- vector(length=2*nrows+1)
for (i in 1:_______________________________) {
   position <- nrows
   for (j in 1:___________________________) {
      if (runif(1) < 0.5) {
         position <- _____________________________
      } else position <- _________________________
   }
   bins[position] <- _____________________________
}
```

3. (10) Fill in the steps, including reasons at the bottom, in the following proof that $Cov(W+R, W-R) = Var(X) - Var(Y)$:

$$
\begin{aligned}
Cov(W+R, W-R) &= \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} && (1)\\
&= Cov(W,W) - Cov(R,R) && (2)\\
&= Var(W) - Var(R) && (3)
\end{aligned}
$$

Reasons (you must cite equation numbers whenever possible, English otherwise):

```
equation (1):
equation (2):
equation (3):
```

4. (15) Suppose $f_X(t)$ is equal to 2t on (0,1), 0 elsewhere. Find P(X > EX), expressing your answer as a fraction, reduced to lowest terms.

5. (10) On p.148, state the standard error of $\hat{c}$.

6. (10) Say $Cov(X) = \Sigma$ for some random vector X. Let v be a nonrandom column vector of the same length as X. Explain clearly why the quantity $v'\Sigma v \geq 0$.

Solutions:

1a. Multinomial, r = 3, n = 5, $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{6}$.

1b. Binomial, n = 5, p = 1/2.

1c. $-6 \cdot \frac{1}{2} \cdot \frac{1}{6}$

1d. Negative binomial, r = 3, p = 1/6.

2. nballs, nrows, position+1, position-1, bins[position]+1

3.

Cov(W,W) - Cov(W,R) + Cov(R,W) - Cov(R,R)

(3.21), algebra and symmetry of Cov(), (3.23)

4.

$$EX = \int_0^1 t \cdot 2t \, dt = \frac{2}{3} \tag{4}$$

$$P(X > EX) = P\left(X > \frac{2}{3}\right) = \int_{2/3}^1 2t \, dt = \frac{5}{9} \tag{5}$$

5. Just use the definitions!

$$\begin{aligned}
s.e.(\hat{c}) &= \text{estimated standard deviation of } \hat{c} \;\; \text{(def. of s.e.)} && (6)\\
&= \sqrt{\widehat{Var(\hat{c})}} \;\; \text{(def. of std. dev.)} && (7)\\
&= \sqrt{\widehat{Var(2\overline{X}+1)}} \;\; \text{(def. of } \hat{c}) && (8)\\
&= \sqrt{4\widehat{Var(\overline{X})}} \;\; \text{(properties. of Var())} && (9)\\
&= 2\sqrt{\widehat{\frac{\sigma^2}{n}}} \;\; ((4.9)) && (10)\\
&= \frac{2}{\sqrt{n}} \cdot \widehat{\sigma} \;\; \text{(alg.)} && (11)\\
&= \frac{2s}{\sqrt{n}} \;\; \text{(def. of s)} && (12)
\end{aligned}$$

A more sophisticated (and possibly a bit more accurate) way to get an estimator of σ^2 would be the write

$$Var(X) = E(X^2) - (EX)^2 = \frac{1}{c}\sum_{i=1}^{c} i^2 - \left(\frac{c+1}{2}\right)^2 \tag{13}$$

One could then plug $\hat{c}$ into this last expression for c, and use the result as $\hat{\sigma}^2$.

6. Let $Y = v'X$. Then

$$\begin{aligned}
v'\Sigma v &= Var(v'X) \;\; ((3.80)) && (14)\\
&= Cov(Y) \;\; \text{(sub)} && (15)\\
&= Var(Y) \;\; \text{(Cov(Y) is a 1x1 matrix consisting of Var(Y)} && (16)\\
&\geq 0 \;\; \text{(variances are nonnegative)} && (17)
\end{aligned}$$