Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing. In order to get full credit, SHOW YOUR WORK.

1. Suppose in running R we have a data set **people**, whose first and second columns contain the heights and ages of the people in our sample, with the third column showing their weights. We will model the regression function of weight on height and age as linear, i.e. $E(\text{weight}|\text{height, age} = \beta_0 + \beta_1 \text{ height} + \beta_2 \text{ age}$.

This question will ask you to write R code, calling **lm()**, etc. For full credit, make efficient use of R (though you are restricted to the parts of R that are in our course). Also, treat the parts below as sequential. For example, if there is code you write for (a) which you also need in (b), treat it as having already been executed when you write your answer for (a).

(a) (15) Give a R code that computes and prints out an approximate 95% confidence interval for β_0.

(b) (15) Give R code that computes and prints out an approximate 95% confidence interval for the mean weight of all people of height 70.2 and age 28.

(c) (15) Give R code that computes and prints out an estimate for σ^2, where we assume that $Var(Y|X = t) = \sigma^2$ for all t.

2. (10) Suppose we roll two dice, with the numbers of dots that come up being denoted by U and V. Let W = U + V. Find $m_{W;U}(2)$. (This can be done intuitively or mathematically.)

3. (20) Suppose the regression of Y on a single predictor variable X, with our model being $m_{Y;X}(t) = \beta_1 t$. There is NO β_0 term in the model, so it is a ONE-parameter family. Assume homogeneous variance as usual, i.e. $Var(Y|X = t) = \sigma^2$ for all t. Find exact variance of $\widehat{\beta}_1$. Your answer will be a function of $X_i^{(1)}$, i = 1,...,n and σ^2.

4. Consider the Markov model of the shared-memory multiprocessor system in our PLN. In each part below, your answer will be a function of $q_1, ..., q_m$.

(a) (10) For the case m = 3, find $p_{(2,0,1),(1,1,1)}$.

(b) (15) For the case m = 6, give a compact expression for $p_{(1,1,1,1,1,1),(i,j,k,l,m,n)}$. (Hint: Some material in a previous PLN solves this problem immediately.)

Bonus Question Suppose that we have a classification setting in which Y = 0,1, and in which given Y = i, X has an exponential distribution with $\lambda = \lambda_i$, i = 0, 1. It is stated in the PLN that the logistic model holds here. Prove this.

Solutions:

1.a

```
lmout <- lm(people[,3] ~ people[,1] + people[,2])
center <- lmout$coefficients[1]
covhat <- vcov(lmout)
se <- sqrt(covhat[1,1])
print(center-1.96*se,center+1.96*se)
```

1.b This was just like the homework problem.

```
a <- c(1,70.2,28)
tmp <- a %*% covhat
se <- tmp %*% a
se <- sqrt(se)
center <- a %*% lmout$coefficients
print(center-1.96*se,center+1.96*se)
```

1.c

```
diffs <- people[,3] - lmout$fitted.values
sumofsquares <- diffs %*% diffs
print(sumofsquares/nrow(people))
```

Some students called **var()** on **diffs**. This actually works, but for reasons far beyond the content of our course.

2. $m_{W;U}(2) = E(W|U = 2)$. This expected value will be 2+EV = 5.5.

3. This is similar to the homework problem.

Since there is no β_0 in the model, there is no 1s column in Q:

$$Q = \begin{pmatrix} X_1^{(1)} \\ X_2^{(1)} \\ ... \\ X_n^{(1)} \end{pmatrix} \tag{1}$$

Then

$$Q'Q = \sum_{i=1}^{n} {X_i^{(1)}}^2 \tag{2}$$

and thus

$$Var[\widehat{\beta_1}] = \frac{\sigma^2}{\sum_{i=1}^{n} {X_i^{(1)}}^2} \tag{3}$$

4.a In state (2,0,1), at the end of the time slot one CPU at Module 1 must choose Module 2 and the CPU at Module 3 must choose Module 3, or vice versa. So, $p_{(2,0,1),(1,1,1)} = 2q_2q_3$.

4.b Guess what, it's a multinomial situation! At the end of the time slot in state (1,1,1,1,1,1), each of the six CPUs will choose a module for its next request. That's six independent trials, with six possible outcomes for each trial, and with the probabilities of those outcomes in a trial being $q_1, ..., q_6$. So, the random vector (i,j,k,l,m,n) has a multinomial distribution with these parameters, and

$$p_{(1,1,1,1,1,1),(i,j,k,l,m,n)} = \frac{6!}{i!j!k!l!m!n!} \tag{4}$$

BQ From the PLN, we know that

$$P(Y = 1|X = t) = \frac{1}{1 + \frac{(1-q) f_{X|Y=0}(t)}{q f_{X|Y=1}(t)}} \tag{5}$$

The right-hand side is equal to

$$\frac{1}{1 + \frac{(1-q)\lambda_0}{q\lambda_1} \cdot e^{(\lambda_0 - \lambda_1)t}} \tag{6}$$

So, we can see that the logistic model holds, with

$$\beta_0 = -\ln(\frac{(1-q)\lambda_0}{q\lambda_1}) \tag{7}$$

and

$$\beta_1 = \lambda_1 - \lambda_0 \tag{8}$$