Name: _____

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**, **sum()**, **integrate()**etc.

1. Consider the good ol' bus ridership examples. Except when referring to the examples in which there is a limit on the number of passengers who can fit into the bus, assume no limit.

(a) (15) Find the probability that in 10 consecutive stops, it turns out that at exactly 3 of them there are no new passengers boarding.

(b) (10) Find Var(T) in (3.134). (Helpful hint: B_1 and B_2 have the same distribution, thus the same variance.)

(c) (10) In Sec. 4.5 (max 20 riders), find p_{12}.

(d) (15) Consider (4.2). The variable **t** there corresponds to what variable in the code in Sec. 2.12.4? (Assume the code has been modified to reflect a 20-rider limit.)

(e) (10) In Sec. 4.5 (max 20 riders), suppose we code the transition matrix in the R matrix **p**. Find $P(L_{28} = 18|L_{25} = 18)$. Your answer must be a valid R expression that involves **p**; no loops.

(f) (10) In Sec. 4.5, suppose the bus is tiny, with a capacity of only 3 passengers. Find the long-run average number of passengers who alight from the bus. Write your answer as a valid R expression in the π vector, which we will assume is named **pivec**. Remember, **pivec[1]** is π_0, etc.

2. (10) Find Var(L) in (3.118).

3. (10) Suppose X has the density te^{-t} on $(0, \infty)$, 0 elsewhere. Find EX. You'll probably want to use the **exp()** function in R.

Solutions:

1.a dbinom(3,10,0.5)

1.b

$$Var(T) = 3^2 Var(B_1) + 2^2 Var(B_2) = 13 Var(B_1) \tag{1}$$

$$Var(B_1) = (0^2 0.5 + 1^2 0.4 + 2^2 0.1) - (0.4 + 2 \cdot 0.1)^2 \tag{2}$$

1.c

$$\begin{aligned} p_{12} &= P(L_{i+1} = 2 | L_i = 1) && (3)\\ &= P(\text{the 1 doesn't alight, 1 new or the 1 alights, 2 new }) && (4)\\ &= (0.8)(0.4) + (0.2)(0.1) && (5) \end{aligned}$$

1.d nstops

1.e

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(p %*% p %*% p)[19,19]
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1.f Mean of binomial is np.

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pivec[2] * (1 * 0.2) + pivec[3] * (2 * 0.2) + pivec[4] * (3 * 0.2)
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2. From (3.117),

$$Var(L) = 3 \cdot \frac{1 - 0.1}{0.1^2} \tag{6}$$

3.

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integrate(function(t) t^2 * exp(-t) ,0,Inf)$value
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