Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed). MAKE SURE TO COPY YOUR ANSWERS TO A SEPARATE SHEET FOR SENDING ME AN ELECTRONIC COPY LATER.

Important note: Remember that in problems calling for R code, you are allowed to use any built-in R function, e.g. **choose()**, **sum()**, etc.

1. Consider the train rendezvous problem in Section 8.2.4. In each of the following, give your answer as a double integral, dt ds. In your electronic answers file, give your answer as five R expressions separated by commas (and optional spaces), as follows: lower limit in outer integral; upper limit in outer integral; lower limit in inner integral; upper limit in inner integral; integrand. For instance, for

$$\int_3^8 \int_s^{1-s} (s+t)^2 \, dt \, ds$$

write

```
3, 8, s, 1-s, (s+t)^2
```

(a) (15) Find $P(X+Y>1.8)$.

(b) (20 Find $F_{X,Y}(0.4, 0.3)$.

2. Suppose the random vector $(X_1, X_2)'$ has mean vector (1.5,2.0)'. Suppose also that each X_i has variance 4 and that $Cov(X_1, X_2) = 1.0$.

(a) (15) Find $E(2X_1 + 3X_2)$, as an R matrix expression.

(b) (15) Find $Var(X_1 + X_2)$, as an R matrix expression.

3. The function below computes the correlation matrix corresponding to a given covariance matrix. Element (i,j) of the latter is the correlation between the ith and the jth elements of the give random vector.

```
covtocorr <- function(covmat) {
   n <- nrow(covmat)
   stddev <- vector(length=n)
   cormat <- matrix(nrow=n,ncol=n)
   for (i in 1:n) {
      stddev[i] <- blank (a)
      cormat[i,i] <- blank (b)
   }
   for (i in 1:(n-1)) {
      for (j in (i+1):n) {
         tmp <- blank (c)
         cormat[i,j] <- tmp
         cormat[j,i] <- tmp
      }
   }
   return(cormat)
}
```

(a) (5) Fill in blank (a).

(b) (5) Fill in blank (b).

(c) (15) Fill in blank (c).

4. (15) Suppose we have three electronic parts, with independent lifetimes that are exponentially distributed with mean 2.5. They are installed simultaneously. Find the mean time until the last failure occurs.

Solutions:

1.a

```
0.8, 1, 1.8-s, 1, 2-s-t
```

1.a

```
0, 0.4, 0, 0.3, 2-s-t
```

2.a

```
c(2,3) %*% c(1.5,2.0)
```

2.b

```
c(1,1) %*% matrix(c(4,1,1,4),nrow=2) %*% c(1,1)
```

3.

```
covtocorr <- function(covmat) {
   n <- nrow(covmat)
   stddev <- vector(length=n)
   cormat <- matrix(nrow=n,ncol=n)
   for (i in 1:n) {
      stddev[i] <- sqrt(covmat[i,i])
      cormat[i,i] <- 1.0
   }
   for (i in 1:(n-1)) {
      for (j in (i+1):n) {
         tmp <- covmat[i,j] / (stddev[i]*stddev[j])
         cormat[i,j] <- tmp
         cormat[j,i] <- tmp
      }
   }
   return(cormat)
}
```

4. As in the computer worm example in Section 8.3.8, the mean is

```
1/(3*0.4) + 1/(2*0.4) + 1/(1*0.4)
```