Name: $\qquad$
Directions: Work only on this sheet (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. (20) Fill in the blank with a term from our course: The value on the left-hand side of (6.1) turns out not to depend on $t$, in the case of $W$ being exponentially distributed. We say that the W has a constant function.
2. Suppose $W_{1}, W_{2}$ and $W_{3}$ are independent, each with distribution $U(0,1)$.
(a) (20) Write (but do not evaluate) an integral for $P\left(W_{1}+W_{2}<0.8\right)$.

In parts (b) and (c), suppose we're interested in finding $\operatorname{Cov}\left(W_{1}+W_{2}, W_{1}+W_{3}\right)$, using (5.107).
(b) (20) Show the matrix A.
(c) (20) Show the matrix $\operatorname{Cov}(\mathrm{W})$.
3. (20) Suppose Pei and Gowtham each take random samples of size 2 with replacement from the three-person population in the toy example on p.185. Find the probability that Gowtham's sample mean is exactly equal to Pei's. EXPRESS YOUR ANSWER AS A SINGLE FRACTION, REDUCED TO LOWEST TERMS, but show your work!

## Solutions:

1. hazard
2.a

$$
\int_{0}^{0.8} \int_{0}^{0.8-s} 1 \cdot 1 d t d s
$$

2.b

$$
\left(\begin{array}{lll}
1 & 1 & 0 \\
1 & 0 & 1
\end{array}\right)
$$

2.c $\mathrm{A} \mathrm{U}(0,1)$ distribution has variance $1 / 12$ and the $W_{i}$ are independent. So the covariance matrix is diagonal, with all diagonal elements equal to $1 / 12$.
3.

$$
\left(\frac{1}{9}\right)^{2}+\left(\frac{2}{9}\right)^{2}+\left(\frac{2}{9}\right)^{2}+\left(\frac{1}{9}\right)^{2}+\left(\frac{2}{9}\right)^{2}+\left(\frac{1}{9}\right)^{2}=5 / 27
$$

