Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. Consider the 2t/15 example, Sec. 4.3.3. Suppose this is the density of light bulb lifetimes L (on the time scale of years). Note: In all parts below, give each answer as decimal expression, e.g. $\sqrt{(0.123)^2 + 5}$, or as a common fraction reduced to lowest terms. You may cite equations in that section.

(a) (20) Find the proportion of bulbs with lifetime less than the mean lifetime.

(b) (20) Find E(1/L).

(c) (20) If I test many bulbs, on average how long will it take to find two that have lifetimes longer than 2.5?

(d) (20) Suppose I've been using bulb A for 2.5 years now in a certain lamp, and am continuing to use it. But at this time I put a new bulb, B, in a second lamp. I am curious as to which bulb is more likely to burn out within the next six months. Find the two probabilities.

2. (20) The expected value of a chi-square random variable with k degrees of freedom turns out to be k. Derive this fact in a step-by-step manner, citing mailing tubes, and NOT using material past p.94.

Solutions:

1.a

$$P(L < 2.8) = \int_1^{2.8} 2t/15 \; dt = (2.8^2 - 1)/15 \quad (1)$$

1.b

$$E(1/L) = \int_1^4 \frac{1}{t} \cdot 2t/15 \; dt = \frac{2}{5} \quad (2)$$

1.c

Use (3.110) with k = 2 and p = 0.65.

1.d First find the probability of NOT burning out in the next six months. For bulb A, use (6.3), yielding

$$\frac{P(L > 3.1)}{P(L > 2.5)} = \frac{\int_{3.1}^4 2t/15 \; dt}{0.65} = \frac{(16 - 3.1^2)/15}{0.65} \quad (3)$$

2. Let Y be as in (4.55). Then

$$\begin{aligned} E(Y) &= E(Z_1^2 + ... + Z_k^2) && (4) \\ &= kE(Z_1^2) \quad (((3.13), \text{ident. distrib.}) && (5) \\ &= k[Var(Z_1) + (EZ_1)^2] \quad ((3,29)) && (6) \\ &= k && (7) \end{aligned}$$