Name: ___________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. Consider the ALOHA simulation on p.47.

(a) (20) On what line do we simulate the possible creation of a new message?

(b) (20) Change line 10 so that it uses **sample()** instead of **runif()**.

2. (20) Say we roll two dice, a blue one and a yellow one. Let B and Y denote the number of dots we get, respectively. Now let G denote the indicator random variable for the event S = 2. Find E(G).

3. Suppose I_1, I_2 and I_3 are independent indicator random variables, with $P(I_j = 1) = p_j$, j = 1,2,3. Find the following in terms of the p_j, writing your derivation in "stacked equation" form [as for example in (3.53)-(3.55)], *with reasons in the form of mailing tube numbers.* You should do reasonable algebraic simplfication of your expressions.

Let $S = I_1 + I_2 I_3$.

(a) (20) ES

(b) (20) Var(S)

Solutions:

1.a 14

1.b

```
numsend <- numsend + sample(0:1,1,prob=c(p,1-p))
```

2. EG = P(G = 1) = P(B+Y = 1) = 1/36

3.a

$$\begin{aligned} ES &= EI_1 + EI_2 \cdot EI_3 \qquad (3.13),\ (3.16) & (1) \\ &= p_1 + p_2 p_3 \qquad (3.43) & (2) \end{aligned}$$

3.b

$$\begin{aligned} Var(S) &= Var(I_1) + Var(I_2 I_3) \qquad (3.64) & (3) \\ &= p_1(1-p_1) + Var(I_2 I_3) \qquad (3.44) & (4) \end{aligned}$$

Let A_j denote the event associated with I_j, j = 1,2,3, and let A denote the event that A_2 and A_3 both occur. Then $I_2 I_3$ is the indicator random variable for A. Thus

$$\begin{aligned} Var(I_2 I_3) &= P(A)[1-P(A)] \qquad (3.44) & (5) \\ &= (p_2 p_3)[1-p_2 p_3] \qquad \text{indep.} & (6) \end{aligned}$$