Name: ----------

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

**1.** (20) In (2.30) and (2.32), cite "mailing tubes" for each, in the form of equation numbers that were used.

**2.** (40) This problem concerns the ALOHA network example. Let $A_i$ denote the event that Node A attempts to transmit during epoch i, i = 1,2,... and define $B_i$ similarly for Node B.

In each case below, you are given two events, in the first two columns of the table. Write in the third column either I, for independent, D, for disjoint, or N, for neither.

| | | |
|---|---|---|
| $A_1$ | $B_1$ | |
| $A_1$ | $B_2$ | |
| $A_1$ | $A_2$ | |
| $A_1$ and $B_1$ | $A_1$ and not $B_1$ | |

**3.** (40) In the simple board game on pp.15ff, let $X_i$ denote your position after your $i^{th}$ turn, i = 1,2,... Find $P(X_1 = 2)$ and $P(X_2 = 2|X_1 = 2)$, giving your answers as fractional expressions, e.g. (1+2/3) / (2 + 1/2).

**Solutions:**

**1.** (2.2), (2.5)

**2.**

| | | |
|---|---|---|
| $A_1$ | $B_1$ | I |
| $A_1$ | $B_2$ | N |
| $A_1$ | $A_2$ | N |
| $A_1$ and $B_1$ | $A_1$ and not $B_1$ | D |

**3.** Let $R_i$ denote your $i^{th}$ ordinary roll, with $B_i$ being the bonus you get for roll i.

$$\begin{aligned} P(X_1 = 2) &= P(R_1 = 2, B_1 = 0 \text{ or } R_1 = 3, B_1 = 6) \\ &= \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} \end{aligned}$$

Get $P(X_2 = 2|X_1 = 2)$ by reasoning it out. After our first turn, if we are at square 2, the only way to be at that square after the next turn is to first roll a 1, getting us to the bonus square 3, and then roll a 6 for our bonus. The probability is thus (1/6) (1/6).