Name: _____

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. (25) Suppose we are studying children's growth patterns, and have data on H_1, H_2 and H_3, heights at ages 6, 10 and 18. We're interested in the growths between 6 and 10, and between 10 and 18, denoted by G_1 and G_2, respectively. Suppose we know the covariance matrix C_H of $(H_1, H_2, H_3)'$. (Assume this to be the population covariance matrix.) Give a *matrix* expression for the covariance matrix of $(G_1, G_2)'$.

2. (25) Consider a toy example in which we take a random sample of size 2 (done with replacement) from a population of size 2. The two values in the population (say heights in some measure system) are 40 and 60. Find $p_{s^2}(100)$. Express your answer as a single common fraction, reduced to lowest terms, but **SHOW YOUR WORK**.

3. (25) Write an R function **bhatcorr(lmobj,i,j)** to act on an object **lmobj** that is returned by **lm()**, with **i** and **j** being subscripts of elements of $\widehat{\beta}$. The function returns the value of

$$\widehat{\rho}(\widehat{\beta}_i, \widehat{\beta}_j)$$

that is, the estimated correlation between $\widehat{\beta}_i$ and $\widehat{\beta}_j$.

4. (25) Consider the example of seek time on p.123. Let $S = |X - Y|$. Find $f_S(v), 0 < v < 1$. Note: Recall that $\int\int_A 1 \; dsdt =$ area(A).

Solutions:

1.

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} C_H \begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}$$

2. Easiest to use (7.18) here. The only way we can get 100 is to sample 40 then 60 or vice versa. The probability is then $2 \cdot (\frac{1}{2})^2 = \frac{1}{2}$.

3.

```
bhatcorr <- function(lmobj,i,j) {
    covmat <- vcov(lmobj)
    vari <- covmat[i+1,i+1]
    varj <- covmat[j+1,j+1]
    covij <- covmat[i+1,j+1]
    return(covij/sqrt(vari*varj)
}
```

4.

$$\begin{aligned} P((X-Y| < v) &= P(-v < X-Y < v) &(1)\\ &= 1-(1-v)^2 &(2)\end{aligned}$$

So, $f_S(v) = 2(1-v)$.