Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

SHOW YOUR WORK!

1. (10) In the ALOHA example, pp.9-12, find $p_{X_1,X_2}(1,2)$. Your answer must consist of a single number found in those pages.

2. Find the following quantities for the density (5.17). **In all cases, do NOT evaluate any integrals; leave your answer in integral form.**

(a) (10) $E(X^2 + XY^{0.5})$

(b) (10) $P(Y > 0.5X)$

(c) (15) $F_{X,Y}(0.6, 0.2)$

3. Find the following quantities for the dice example on p.115. **In all cases, leave your answers as numerical expressions, e.g.** $5/2.3+6^{1+\sqrt{2}}$. Feel free to cite (3.29).

(a) (10) Cov(X,2S)

(b) (10) Cov(X,S+Y)

(c) (10) Cov(X+2Y,3X-Y)

(d) (10) $p_{X,S}(3,8)$

4. (15) Consider the "Senthi" example, pp.118-119. Let R denote the time it takes to go from state 1 to state 3. Find $f_R(v)$. **Leave your answer in integral form.**

Solutions:

1. 0.20 (see just below (2.14))

2a.

$$\int_0^1 \int_0^s (s^2 + st^{0.5}) \ 8st \ dt \ ds$$

2b.

$$\int_0^1 \int_{0.5s}^s 8st \ dt \ ds$$

2c. By definition,

$$F_{X,Y}(0.6, 0.2) = P(X \leq 0.6 \text{ and } Y \leq 0.2)$$

The region in question has an irregular shape, so the so the answer is a two-part integral:

$$\int_0^{0.2} \int_0^s 8st \ dt \ ds + \int_{0.2}^{0.6} \int_0^{0.2} 8st \ dt \ ds$$

3a. This and the next two parts make use of (5.22) and other mailing tubes.

$$Cov(X, 2S) = 2Cov(X, S) = 2 \cdot 0.707$$

using (5.72).

3b.

$$Cov(X, S + Y) = Cov(X, S) + Cov(X, Y) = 0.707 + 0$$

3c. In (5.22), take a = 1, b = 2, c = 3 and d = -1. Then use the fact that Cov(X,X) = Var(X), etc. The result is

$$3Var(X) - 2Var(Y) = Var(X) = 2.92$$

4. Write $R = R_1 + R_2$, where R_i is the time to go from state i to state i+1. R_1 and R_2 are independent, due to the Markov/memoryless property, and as the example points out, R_i has an exponential distribution with parameter i(g-i). So, we are basically in the same situation as the backup battery example in Sec. 5.5.4, and

$$f_R(v) = \int_0^v \lambda_1 e^{-\lambda_1 s} \lambda_2 e^{-\lambda_2 (v-s)} \ ds$$

where $\lambda_i = i(g - i)$.