Name: ___________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

SHOW YOUR WORK!

1. (20) Suppose we roll our usual three-sided die, with probabilities 1/2, 1/3 and 1/6 of coming up 1, 2 or 3 dots, respectively. Let X denote the number of dots. Find $h_X(2)$. Express your answer as a single common fraction.

2. (25) Write R code (but not simulation) that computes the value of

$$\int_{27}^{30} \frac{1}{\sqrt{2\pi}\cdot 5}\ e^{-0.5\left(\frac{t-28}{5}\right)^2}\ dt$$

3. (25) Consider the disk performance example on p.76. We will scale things so that the track number is a continuous value in [0,1]. Fill in the gaps in the following code, which finds the (approximate) mean time to satisfy a disk access request. The arguments **fullsweep** and **fullrotate** are the time needed to go from track 0.0 to track 1.0, and the time needed to make one revolution of the disk, respectively.

```
disksim <- function(naccesses,fullsweep,fullrotate) {
   currtrack <- 0.5
   oldtrack <- 0.5
   sumacctime <- 0.0
   for (i in 1:naccesses) {
      currtrack <-                                # gap
      seek <- abs(currtrack - oldtrack)
      oldtrack <- currtrack
      seektime <-                                 # gap
      rottime <-                                  # gap
      sumacctime <-                               # gap
   }
   return(                 )                      # gap
}
```

4. Consider the following variant of the bus ridership example on p.20 and our current homework. The probability that a passenger alights is now q instead of 0.2, and the number of new passengers who wish to board the bus at a stop, N, is now assumed to have a Poisson distribution with parameter λ. The capacity of the bus is still c. Answer the following, using expressions in c, q, λ and the stationary probability vector π (you may not need them all).

(a) (15) Find the transition probabilities p_{00} and p_{21}.

(b) (15) Let S denote the number of stops that a passenger travels. If for instance she boards at stop 3 and alights at stop 8, then S = 5. Find Var(S)

Solutions:

1.

$$h_X(2) = \frac{p_X(2)}{1-F_X(1)} = \frac{1/3}{1-1/2} = 2/3$$

2.

```
pnorm(30,mean=28,sd=5) - pnorm(27,mean=28,sd=5)
```

3.

```
disksim <- function(naccesses,fullsweep,fullrotate) {
   currtrack <- 0.5
   oldtrack <- 0.5
   sumacctime <- 0.0
   for (i in 1:naccesses) {
      currtrack <- runif(1)
      seek <- abs(currtrack - oldtrack)
      oldtrack <- currtrack
      seektime <- seek * fullsweep
      rottime <- runif(0,fullrotate)
      sumacctime <- seektime + rottime
   }
   return(sumacctime/naccesses)
}
```

4.a For p_{00}, that transition will occur if 0 board, which has probability

$$\frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda}$$

For the case p_{21}, either 1 alights and 0 board or 2 alight and 1 boards, so we have

$$2q(1-q)\frac{\lambda^0 e^{-\lambda}}{0!} + q^2 \cdot \frac{\lambda^1 e^{-\lambda}}{1!} = qe^{-\lambda}[2(1-q)+q\lambda] = qe^{-\lambda}[2+(\lambda-2)q]$$

4.b S has a geometric distribution with parameter q, so by (3.75), $Var(S) = (1-q)/q^2$.