

Name: \_\_\_\_\_

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

## SHOW YOUR WORK!

- (15) Exercise 7(b), Chapter 4, p.97. Give your answer as a decimal or common fraction.
- (15) Exercise 8(a), Chapter 4, p.97. Give your answers as decimal or common fractions.
- (20) Suppose X has a uniform distribution on the interval (20,40), and we know that X is greater than 25. What is the probability that X is greater than 32? Give your answer as a common fraction.
- (25) Suppose U and V have the  $2t/15$  density on p.74. Let N denote the number of values among U and V that are greater than 1.5. (CORRECTED SUBSEQUENT TO QUIZ.) (So N is either 0, 1 or 2.) Find  $\text{Var}(N)$ , expressing your answer as a decimal or common fraction.
- (25) What is the (approximate) value returned from the following R code?

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mean((rnorm(10000,mean=28,sd=5))^4)
```

Your answer must be expressed as a definite integral.

### Solutions:

1

$$F_X(0.2) = \int_0^{0.2} 2(1-t) dt = 0.36$$

$$EX = \int_0^1 t \cdot 2(1-t) dt = 1/3$$

- Let X be the error. On p.75, we have  $r = 0.5$ ,  $q = -0.5$ . Using the formulas for the mean and variance at the bottom of p.75, we have

$$E(X) = (q + r)/2 = 0$$

$$\text{Var}(X) = (r - q)^2/12 = 1/12$$

- Because of the uniformity,  $P(a < X < b) = (b-a)/20$ . Following the pattern on p.79, we have

$$P(X > 30 | X > 25) = \frac{P(X > 30)}{P(X > 25)} = \frac{10/20}{15/20} = 2/3$$

- N has a binomial distribution with  $n = 2$  and

$$p = \int_{1.5}^4 2t/15 dt = \frac{11}{12}$$

So, (once again) using (3.82), we have

$$\text{Var}(N) = 2 \cdot \frac{11}{12} \cdot \frac{1}{12} = \frac{11}{72}$$

- The simulation is calculating  $E(X^4)$ , where X has a normal distribution with mean 28 and standard deviation 5. That expected value, by (4.21), is

$$\int_{-\infty}^{\infty} t^4 \cdot \frac{1}{\sqrt{2\pi} \cdot 5} e^{-0.5\left(\frac{t-28}{5}\right)^2} dt$$