Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing. **SHOW YOUR WORK!** Any arithmetical answer must be expressed as a common fraction (e.g. 2/3, 7/4), reduced to lowest terms.

1. (25) Urn I contains three blue marbles and three yellow ones, while Urn II contains five and seven of these colors. We draw a marble at random from Urn I and place it in Urn II. We then draw a marble at random from Urn II. Let B_1 denote the event that the first marble drawn is blue, Y_2 denote the event that the second marble drawn is yellow, and so on. Fill in the blanks with equation numbers which will serve as reasons for the steps,

$$
\begin{aligned}
P(B_2) &= P(B_1 \text{ and } B_2 \text{ or } Y_1 \text{ and } B_2) && (1)\\
&= P(B_1 \text{ and } B_2) + P(Y_1 \text{ and } B_2) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} && (2)\\
&= P(B_1)P(B_2|B_1) + P(Y_1)P(B_2|Y_1) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} && (3)
\end{aligned}
$$

2. (25) Fill in the blanks (and <u>only</u> those blanks, no extra code elsewhere) in the following R code, which returns the (approximate) probability in (2.36) in the board game example:

```
boardsim <- function(nreps) {
   count4 <- 0
   countbonusgiven4 <- 0
   for (i in 1:                              ) {   # blank 1
      position <- sample(1:6,1)
      if (position == 3) {
                                                   # blank 2
         position <- (position + sample(1:6,1)) %% 8
      }                                            # blank 3
      if (position == 4) {
                                                   # blank 4
         if (bonus) countbonusgiven4 <- countbonusgiven4 + 1
      }
   }
   return(                                 )      # blank 5
}
```

3. (25) Suppose the random variable X takes on only the values 0 and 1. Fill in the blank with either $<$, $\leq$, $=$, $\neq$, $\geq$ or *no relation*:

EX   P(X = 1)

4. (25) Again for the board game example, suppose that the telephone report is that A ended up at square 1 after his first turn. Find the probability that he got a bonus.

Solutions:

1. (2.2), (2.5)

2.

```
boardsim <- function(nreps) {
   count4 <- 0
   countbonusgiven4 <- 0
   for (i in 1:nreps) {
      position <- sample(1:6,1)
      if (position == 3) {
         bonus <- TRUE
         position <- (position + sample(1:6,1)) %% 8
      } else bonus <- FALSE
      if (position == 4) {
         count4 <- count4 + 1
         if (bonus) countbousngiven4 <- countbousngiven4 + 1
      }
   }
   return(countbousngiven4/count4)
}
```

3. The answer is =, since

$$EX = 1 \cdot P(X=1) + 0 \cdot P(X=0) = P(X=1)$$

4. Landing at square 1 after one turn means R+B is either 1 or 9. Let T = R + B.

$$\begin{aligned}
P(B>0|T=1 \text{ or } T=9) &= \frac{P((T=1 \text{ or } T=9) \text{ and } B>0)}{P(T=1 \text{ or } T=9)} && (4)\\
&= \frac{P((R,B)=(3,6))}{P((R,B)=(1,0) \text{ or } (R,B)=(3,6))} && (5)\\
&= \frac{\frac{1}{6}\cdot\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}} && (6)\\
&= \frac{1}{7} && (7)
\end{aligned}$$