Name: ____________________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

**1.** Suppose X has an $N(\mu, \mu^2)$ distribution, i.e. with the standard deviation equal to the mean. (A common assumption in regression contexts.) Show that $h(X) = \ln(X)$ will be a variance-stabilizing transformation.

**2.** For each of the following quantities, state whether the given estimator is unbiased in the given context:

(a) (4.15), p. 97, as an estimator of $\sigma^2$

(b) $\hat{p}$, as an estimator of p, p.105

(c) $\hat{p}(1-\hat{p})$, as an estimator of p(1-p), p.105

(d) $\bar{X} - \bar{Y}$, as an estimator of $\mu_1 - \mu_2$, p.107

(e) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \mu_1)^2$ (assuming $\mu_1$ is known), as an estimator of $\sigma_1^2$, p.107

(f) $\bar{X}$, as an estimator of $\mu_1$, p.107, *but sampling (from the population of Davis) without replacement*

**3.** Consider the classical, unbiased estimator of $\sigma^2$,

$$\tilde{s}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(W_i - \overline{W})^2 \tag{1}$$

Show that $\tilde{s}$ is a biased estimator of $\sigma$. (Hint: Consider the quantity $Var(\tilde{s})$.)

**Solutions:**

**1.** $d/dt\ ln(t) = 1/t$, so

$$\text{asymp.}Var[h(X)] = [h(\mu]^2 Var(X) = (1/\mu)^2\mu^2 = 1 \tag{2}$$

**2.**

(a) biased, as shown in Section 4.2.7

(b) unbiased, since $\hat{p}$ is a special case of $\bar{W}$, and the latter was shown to be unbiased in Section 4.2.2.1

(c) biased, since $\hat{p}(1-\hat{p})$ is a special case of $s^2$; see (a) above

(d) unbiased, again due to Section 4.2.2.1

(e) using the properties of expected value and the fact that each $X_i$ has the same distribution as population 1, we have that the expected value of the specified quantity is $E[(X-\mu_1)^2$, i.e. $\sigma_1^2$, and thus the quantity is unbiased

(f) unbiased, since the derivation of $E(\bar{W}) = \mu$ in Section 4.2.2.1 used only properties of E() that do not require independence

**3.** (This problem was intended to be difficult, though short.)

$$\begin{aligned} 0 &< Var(\tilde{s}) \quad (\tilde{s} \text{ is not const.}) && (3)\\ &= E(\tilde{s}^2) - [E(\tilde{s})]^2 \quad (1.44) && (4)\\ &= \sigma^2 - [E(\tilde{s})]^2 \quad \tilde{s} \text{ is unbiased} && (5) \end{aligned}$$

Therefore $[E(\tilde{s})] < \sigma$, and $[\tilde{s}]$ is biased, in fact biased downward.