Name: ____________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

**Unless otherwise stated, give numerical answers as expressions, e.g. \( \frac{2}{3} \times 6 - 1.8 \). Do NOT use calculators.**

1. (20) Fill in the blank: Density functions for continuous random variables are analogs of the ________________ functions that are used for discrete random variables.

2. (20) Suppose for some random variable \( W \), \( F_W(t) = t^3 \) for \( 0 < t < 1 \), with \( F_W(t) \) be 0 and 1 for \( t < 0 \) and \( t > 0 \), respectively. Find \( f_W(t) \) for \( 0 < t < 1 \).

3. (10) Suppose \( X \) has a binomial distribution with parameters \( n \) and \( p \). Then \( X \) is approximately normally distributed with mean \( np \) and variance \( np(1-p) \). For each of the following, answer either A or E, for “approximately” or “exact,” respectively:
   
   (a) (10) distribution of \( X \) is normal 
   (b) (10) \( E(X) \) is \( np \) 
   (c) (5) \( \text{Var}(X) \) is \( np(1-p) \)

4. Suppose light bulb lifetimes have an exponential distribution with mean 100.0 hours, i.e. \( \lambda = 0.01 \). We use our first light bulb, with it lasting for \( X_1 \) hours. When it burns out, we replace it with a second bulb, which lasts \( X_2 \) hours. Then \( T_2 = X_1 + X_2 \) is the time of the second replacement.

   (a) (10) Give numerical expressions for the mean and variance of \( T_2 \).
   (b) (5) State \( f_{T_2}(t) \) (the actual function, not the name of a family etc.).
   (c) (10) Fill in the blank:

\[
P(T_2 > t) = P[N(t)_____________]\]

**Solutions:**
1. probability mass functions
2. 3\( \bar{t}^2 \)
3a. A (pages 30-31)
3b. E (pages 30-31)
3c. E (pages 30-31)
4a. \( 2 \cdot 100, 2 \cdot 100^2 \) (page 59)
4b. \( 0.01^2te^{-\lambda t} \) (page 59)
4c. \( \leq 1 \) (like (2.38))