Name: ______________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. Consider the $2t/15$ example, Sec. 4.3.3. Suppose this is the density of light bulb lifetimes $L$ (on the time scale of years). Note: In all parts below, give each answer as decimal expression, e.g. $\sqrt{(0.123)^2 + 5}$, or as a common fraction reduced to lowest terms. You may cite equations in that section.

(a) (20) Find the proportion of bulbs with lifetime less than the mean lifetime.

(b) (20) Find $E(1/L)$.

(c) (20) If I test many bulbs, on average how long will it take to find two that have lifetimes longer than 2.5?

(d) (20) Suppose I’ve been using bulb A for 2.5 years now in a certain lamp, and am continuing to use it. But at this time I put a new bulb, B, in a second lamp. I am curious as to which bulb is more likely to burn out within the next six months. Find the two probabilities.

2. (20) The expected value of a chi-square random variable with $k$ degrees of freedom turns out to be $k$. Derive this fact in a step-by-step manner, citing mailing tubes, and NOT using material past p.94.

**Solutions:**

1.a

\[ P(L < 2.8) = \int_{1}^{2.8} \frac{2t}{15} dt = \frac{(2.8^2 - 1)}{15} \quad (1) \]

1.b

\[ E(1/L) = \int_{1}^{4} \frac{1}{t} \cdot \frac{2t}{15} dt = \frac{2}{5} \quad (2) \]

1.c

Use (3.110) with $k = 2$ and $p = 0.65$.

1.d First find the probability of NOT burning out in the next six months. For bulb A, use (6.3), yielding

\[ \frac{P(L > 3.1)}{P(L > 2.5)} = \frac{\int_{3.1}^{4} \frac{2t}{15} dt}{0.65} = \frac{(16 - 3.1^2)/15}{0.65} \quad (3) \]

2. Let $Y$ be as in (4.55). Then

\[
E(Y) = E(Z_1^2 + \ldots + Z_k^2) \\
= kE(Z_1^2) \quad (((3.13), \text{ident. distrib.}) \quad (5) \\
= k[Var(Z_1) + (EZ_1)^2] \quad ((3, 29)) \quad (6) \\
= k \quad (7)
\]