1. Consider the ALOHA simulation on p.47.

(a) (20) On what line do we simulate the possible creation of a new message?

(b) (20) Change line 10 so that it uses \texttt{sample()} instead of \texttt{runif()}.

2. (20) Say we roll two dice, a blue one and a yellow one. Let \( B \) and \( Y \) denote the number of dots we get, respectively. Now let \( G \) denote the indicator random variable for the event \( S = 2 \). Find \( E(G) \).

3. Suppose \( I_1, I_2 \) and \( I_3 \) are independent indicator random variables, with \( P(I_j = 1) = p_j, j = 1,2,3 \). Find the following in terms of the \( p_j \), writing your derivation in “stacked equation” form [as for example in (3.53)-(3.55)], \textit{with reasons in the form of mailing tube numbers}. You should do reasonable algebraic simplification of your expressions.

Let \( S = I_1 + I_2I_3 \).

(a) (20) \( ES \)

(b) (20) \( \text{Var}(S) \)
Solutions:

1.a 14

1.b

numsend <- numsend + sample(0:1,1,prob=c(p,1-p))

2. \( EG = P(G = 1) = P(B + Y = 1) = 1/36 \)

3.a

\[
ES = EI_1 + EI_2 \cdot EI_3 \quad (3.13), (3.16) \\
= p_1 + p_2 p_3 \quad (3.43)
\]

3.b

\[
Var(S) = Var(I_1) + Var(I_2 I_3) \quad (3.64) \\
= p_1 (1 - p_1) + Var(I_2 I_3) \quad (3.44)
\]

Let \( A_j \) denote the event associated with \( I_j \), \( j = 1,2,3 \), and let \( A \) denote the event that \( A_2 \) and \( A_3 \) both occur. Then \( I_2 I_3 \) is the indicator random variable for \( A \). Thus

\[
Var(I_2 I_3) = P(A)[1 - P(A)] \quad (3.44) \\
= (p_2 p_3)[1 - p_2 p_3] \quad \text{indep.} \ (6)
\]