1. (25) Suppose we are studying children’s growth patterns, and have data on \( H_1, H_2 \) and \( H_3 \), heights at ages 6, 10 and 18. We’re interested in the growths between 6 and 10, and between 10 and 18, denoted by \( G_1 \) and \( G_2 \), respectively. Suppose we know the covariance matrix \( C_H \) of \((H_1, H_2, H_3)'\). (Assume this to be the population covariance matrix.) Give a matrix expression for the covariance matrix of \((G_1, G_2)'\).

2. (25) Consider a toy example in which we take a random sample of size 2 (done with replacement) from a population of size 2. The two values in the population (say heights in some measure system) are 40 and 60. Find \( p_{s^2}(100) \). Express your answer as a single common fraction, reduced to lowest terms, but SHOW YOUR WORK.

3. (25) Write an R function \texttt{bhatcorr(lmobj,i,j)} to act on an object \texttt{lmobj} that is returned by \texttt{lm()}, with \( i \) and \( j \) being subscripts of elements of \( \hat{\beta} \). The function returns the value of

\[
\hat{\rho}(\hat{\beta}_i, \hat{\beta}_j)
\]

that is, the estimated correlation between \( \hat{\beta}_i \) and \( \hat{\beta}_j \).

4. (25) Consider the example of seek time on p.123. Let \( S = |X - Y| \). Find \( f_S(v), 0 < v < 1 \). Note: Recall that \( \int \int_A 1 \, dsdt = \text{area}(A) \).
Solutions:

1. 
   \[
   \begin{pmatrix}
   -1 & 1 & 0 \\
   0 & -1 & 1
   \end{pmatrix}
   \begin{pmatrix}
   -1 & 0 \\
   1 & -1 \\
   0 & 1
   \end{pmatrix}
   \]

2. Easiest to use (7.18) here. The only way we can get 100 is to sample 40 then 60 or vice versa. The probability is then \(2 \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2}\).

3. 
   \[\text{bhatcorr} \leftarrow \text{function}(\text{lmobj}, i, j) \{ \right.\]
   \[
   \text{covmat} \leftarrow \text{vcov}(\text{lmobj})
   \]
   \[
   \text{vari} \leftarrow \text{covmat}[i+1, i+1]
   \]
   \[
   \text{varj} \leftarrow \text{covmat}[j+1, j+1]
   \]
   \[
   \text{covij} \leftarrow \text{covmat}[i+1, j+1]
   \]
   \[
   \text{return}(\text{covij}/\sqrt{\text{vari} \ast \text{varj}})
   \}
   \]

4. 
   \[
P((X - Y) < v) = P(-v < X - Y < v)
   \]
   \[
   = 1 - (1 - v)^2
   \]

So, \(f_S(v) = 2(1 - v)\).