Name: ______________________

Directions: **Work only on this sheet** (on both sides, if needed); do not turn in any supplementary sheets of paper. There is actually plenty of room for your answers, as long as you organize yourself BEFORE starting writing.

1. X be the number of dots we get in rolling a three-sided die once. (It’s cylindrical in shape.) The die is weighted so that the probabilities of one, two and three dots are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$, respectively. Note: Express all answers in this problem as common fractions, reduced to lowest terms, such as $\frac{2}{3}$ and $\frac{9}{7}$.

(a) (10) State the value of $p_X(2)$.

(b) (10) Find $EX$ and $Var(X)$.

(c) (15) Suppose you win $2 for each dot. Find $EW$, where $W$ is the amount you win.

2. This problem concerns the REVISED version of the committee/gender example.

(a) (10) Find $E(D^2)$. Express your answer as an unsimplified expression involving combinatorial quantities such as $\binom{168}{28}$.

(b) (15) Find $P(G_1 = G_2 = 1)$. Express your answer as a common fraction.

3. (15) State the (approximate) return value for the function below, in terms of $w$. You must cite an equation number in the book to get full credit.

```r
xsim <- function(nreps, w) {
    sumn <- 0
    for (i in 1:nreps) {
        n <- 0
        while (TRUE) {
            n <- n + 1
            u <- runif(1)
            if (u < w) break
        }
        sumn <- sumn + n
    }
    return(sumn/nreps)
}
```

4. (15) Consider the parking space example on p.48. (NOT the variant in the homework.) Let $N$ denote the number of empty spaces in the first block. State the value of $Var(N)$, expressed as a common fraction.

5. (10) Suppose $X$ and $Y$ are independent, with variances 1 and 2, respectively. Find the value of $c$ that minimizes $Var[cX + (1-c)Y]$.

**Solutions:**

1.a $\frac{1}{3}$

1.b

\[ EX = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{6} = \frac{5}{3} \]

\[ Var(X) = E(X^2) - (EX)^2 \]

\[ = 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{3} + 3^2 \cdot \frac{1}{6} - \frac{5}{9} \]

\[ = \frac{5}{9} \]

1.c $EW = E(2X) = 2 \cdot EX = \frac{10}{3}$

2.a

\[ E(D^2) = (-2)^2 \frac{\binom{168}{6}}{\binom{28}{3}} + ... \]

2.b

\[ P(G_1 = G_2 = 1) = P(G_1 = 1)P(G_2 = 1|G_1 = 1) \]

\[ = \frac{6}{9} \cdot \frac{5}{8} \]

\[ = \frac{5}{12} \]

3. $\frac{1}{w}$, by (3.74)

4. $10(0.2)(1-0.2) = \frac{8}{5}$, by (3.82)

5.

\[ 0 = \frac{d}{dc} Var[cX + (1-c)Y] \]

\[ = \frac{d}{dc} [c^2 Var(X) + (1-c)^2 Var(Y)] \]

\[ = \frac{d}{dc} [c^2 + 2(1-c)^2] \]

\[ = 2c - 4(1-c) \]

So, the best $c$ is $\frac{2}{3}$. 