$$a_n = \frac{2}{T}\int_0^T x(t)cos(2\pi n f_0 t)dt \tag{8}$$

$$b_n = \frac{2}{T}\int_0^T x(t)sin(2\pi n f_0 t)dt \tag{9}$$

Because of the periodic nature of the functions involved, we can shift the range of integration by equal amounts on the lower and upper bounds, and it is often convenient to do so if we are calculating the integrals by hand. Any lower and upper bounds which differ by the amount T will give the same answer.

We say that x(t) is the **energy level** of the signal. For example, if x(t) is a graph of your voice over time, x(t) is the loudness of your voice at time t. The a_n and b_n then show how the energy of the signal break down into different frequencies; in fact, the average squared energy of the signal is the sum of the squares of these coefficients:

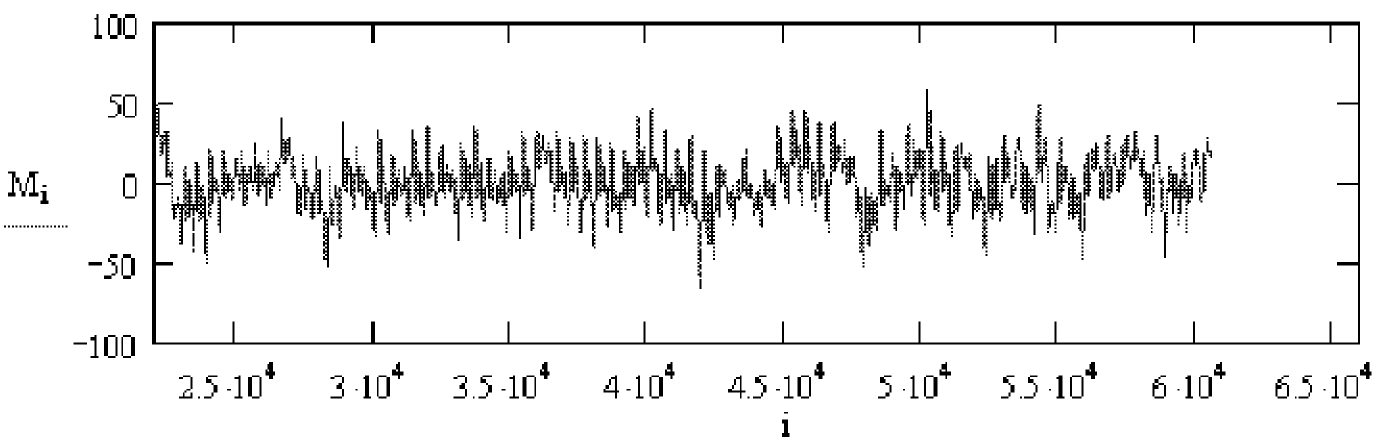
$$\frac{1}{T}\int_0^T x^2(t)dt = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

We say that x(t) is the **time domain** version of the signal, and a_n and b_n comprise the **frequency domain**.

We can also write x as an integral of trig functions, rather than a sum of such functions. Then the spectrum is a continuous range of numbers, rather than the discrete points a_n and b_n. This is called the **Fourier Transform** of the original periodic function.

3.2 Example: Time- and Frequency-Domain Graphs for a Vibrating Reed

Here is a time-domain graph of the sound made by a vibrating reed:[1]



[1] Reproduced here by permission of Prof. Peter Hamburger, Indiana-Purdue University, Fort Wayne. See http://www.ipfw.edu/math/Workshop/PBC.html