Dealing with “And”, “Or” in Probability Computations: a Review Day

Example:

A school holds a raffle and sells 20 tickets. You bought a block of 4 tickets, numbered 15 to 18. Two tickets will be drawn at random.

- What are the chances you win two prizes?
- What is the probability that you don’t win on the first ticket drawn but you do win on the second?
- What is the probability that you win at least one prize?

Let's work on that first problem, the probability of winning two prizes.

First, a new concept (new last week):

In addition to ordinary probabilities, we can also talk about **conditional** probabilities, such as

P(win on second ticket | win on first ticket)

That vertical bar, |, is pronounced "given." We read

P(win on second ticket | win on first ticket)

aloud as "the probability that we win on the second ticket given that we win on the first ticket." It means this:

Say they have just drawn the first ticket, and you see that it is is one of yours — its number is between 15 and 18.

Now there are 19 tickets left, 3 of which are yours. So,

P(win on second ticket | win on first ticket) = $\frac{3}{19}$

“Notebook interpretation”:

P(win on second ticket | win on first ticket) = $\frac{3}{19}$

means that if you do the experiment, say, 10,000 times, then **among** those notebook lines in which you win on the first ticket, 3/19 of **those** lines will have you winning on the second ticket.

On the other hand, P(win on second ticket) — with no condition — is the fraction of lines in which you win on the second ticket, among **all** the notebook lines.

Reminder: Our discussions using “notebooks” are just so we can understand what probabilities *mean*. They are not useful as tools to *solve* the problem. So, what *can* we use?

Handy Rule #1:

Say A and B are two "events" involving an experiment.
Then P(A and B) = P(A) $\times$ P(B | A).

Can we use this to find P(win on first ticket and win on second ticket)?

In other words, are there useful choices of A and B here? Yes:

A = "win on first ticket"
B = "win on second ticket"

So

P($\boxed{\text{win on first ticket}}$ AND $\boxed{\text{win on second ticket}}$) =
P(win on first ticket) $\times$ P(win on second ticket | win on first ticket) =
$\frac{4}{20} \times \frac{3}{19} = \frac{3}{95}$

Now, how do we find, say, P(win on exactly one ticket)?

Handy Rules #2 and 3:

> "Break big events into small events."
>
> That means to break the event you have into a bunch of "and" and "or" sub-events.
>
> Also, if A and B don't overlap,
>
> P(A or B) = P(A) + P(B)

So,

P(win on exactly one ticket) =

P($\boxed{\text{win on first and lose on second}}$ OR $\boxed{\text{lose on first and win on second}}$) =

P(win on first ticket and lose on second) + P(lose on first and win on second) =

_______ × _______ + _______ × _______ =

In-class exercise:

Fill in the 4 blanks above to finish that problem. To do this, note that we have "and"s here, so we can just follow the same pattern we saw in our "and" example on the preceding page.

Handy Hint #4

P(A) = 1 - P(not A)

Sometimes P(not A) is easier to find than P(A); then you just subtract.

Example: Find P(win at least one prize).

We could break it down into three cases (win,win; win,lose; lose,win), but it's easier to use Hint #4:

P(win at least one prize)

= 1 - P(win no prize)

= 1 - P(lose on first and lost on second)

= 1 - $\frac{16}{20} \times \frac{15}{19}$

= $\frac{12}{19}$

$\approx \frac{2}{3}$

Handy Hint #5

> With a string of "and"s, you can keep multiplying.
>
> For example, if you have three events, A, B and C, then
>
> P(A and B and C) = P(A) $\times$ P(B|A) $\times$ P(C|A,B)

Example:

> In our earlier lottery example, suppose three tickets are drawn, not two. Find P(win on first and lose on second and win on third).

To find this probability, just choose A to be "win on first," B to be "lose on second" and C to be "win on third." Then you see that

P(win on first and lose on second and win on third) =

$\frac{4}{20} \times \frac{16}{19} \times \frac{3}{18} = \frac{8}{285}$.

Homework:

Main problem: In the version of the lottery example in which three tickets are drawn, find the probability that you win at least one prize.

Challenge problem: In last week's jelly bean example (the original version, not the one in the homework), find

P(the bean you get from II was originally in I | the bean you get from II is purple)

Hint: Say you have three numbers, a, b and c. Then if $a \times b = c$, then $b = c \div a$.

Next week:

We will finally be able to solve the poker-hand problem.