# Introductory Example: How Do Gambling Casinos Make Money?

A simple game:

Roll one die (*dice* is the plural of *die*).

Here is what you win:

| if you roll... | then you win |
|:---:|:---:|
| 1, 2 or 3 | 0 |
| 4 or 5 | $0.75 |
| 6 | $12.00 |

**Question:** How much should the casino charge players to play this game?

Watch what happens as more and more people play the game:

Say 600 people have played the game so far. Then:

- $\approx 300$ people have won 0

- $\approx 200$ people have won $0.75

- $\approx 100$ people have won $12.00

- so the casino has paid out $\approx 200 \times \$0.75 + 100 \times \$12.00$

So, the casino has paid out about $1,350, for 600 players.

So, how much should the casino have charged as a fee to play this game, in order for the casino to not lose money?

How about charging $1? No, then the casino would have taken in $600 \times \$1 = \$600$, while paying out $1,350 — the casino would go out of business!

How about charging $3? Then the casino takes in $1,800 and doesn't lose money, but a lot of people may consider $3 too expensive, and not play anymore.

To just "break even," the casino would need to charge

$$\frac{\$1,350}{600} = \$2.25$$

(Then maybe charge a bit more than this, to cover other costs and add profit, etc.)

What we did above was calculate something called *expected value*. We'll return to this at a later date, but right now let's focus on the relation of expected value to probability.

We use the notation P( ) to mean "probability of." For example, if you flip a coin, we say

$$P(heads) = \frac{1}{2}$$

If you go back to our analysis of the casino, you will find that we made use of the following probabilities:

$$P(1, 2 \ or \ 3) = \frac{1}{2}$$

$$P(4 \ or \ 5) = \frac{1}{3}$$

$$P(6) = \frac{1}{6}$$

So, probabilities are key to the casinos. Thus we will concentrate on probability for now, and return to expected value later.

Homework:

(We will <u>not</u> have a lot of homework, but we do need a little bit each week, so that we don't forget the concepts. Hand in your solution in written form. You may work with one partner if you wish, in which case turn in just one paper with both of your names.)

The game is to toss a coin 3 times. The player wins \$20 for 3 heads, \$8 for 2 heads, and 0 otherwise. Determine how much the casino must charge to just break even.

Hint: Use a 3-letter notation for the outcome. TTT means all 3 flips result in tails; TTH means the first two flips are tails and then the third flip is heads; THT means the player gets a tail, then a head, then a tail; etc., with 8 of these patterns in all.

Next week:

# The Totally-Magical-and-All-Powerful-But-Seemingly-Nothing-to-It Principle of Probability

This is secret stuff! No book and no other teacher will tell you this, but it is extremely powerful.